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A Note on the Wind Stress over Disturbed Sea Surface

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1. Consider the turbulent boundary layer over sea surface in which the vertical eddy transfer of lateral vorticity is constant.

$$\frac{d\tau_e}{dz} + \frac{d\tau_\omega}{dz} = \text{const} \tag{1}$$

where τ_e denotes the turbulent Reynolds stress and τ_ω the Reynolds stress associated with the surface waves.

According to the new vorticity-transfer hypothesis of turbulence theory by Lettau (LETTAU, H. 1964) we have

$$\begin{aligned} \frac{1}{\rho}\frac{d^2\tau_e}{dz^2} &= \frac{d}{dz}\left(-\overline{\omega\eta}\right) \\ &= \frac{\partial}{\partial z}\left[K\frac{\partial^2 U}{\partial z^2}\right] - \kappa^2\left(\frac{\partial U}{\partial z}\right)^2 \end{aligned} \tag{2}$$

where κ is Kármán constant and K is eddy diffusivity.

Invoking the well known similarity hypothesis, we shall assume that

$$K = l u_* \quad , \ l = -\kappa \frac{\frac{\partial U}{\partial z}}{\frac{\partial^2 U}{\partial z^2}} \tag{3}$$

Differentiating eq. (1) with z and substituting eq. (2) and eq. (3) we have in non-dimensional form

$$\frac{\partial^2 U}{\partial z^2}\left(1 - \frac{\partial \tau_\omega}{\partial U}\right) + \left(\frac{\partial U}{\partial z}\right)^2\left(1 - \frac{\partial^2 \tau_\omega}{\partial U^2}\right) = 0 \tag{4}$$

where U is nondimensionalized by $u_1 = \frac{u_*}{\kappa}$, z by z_0, τ by ρu_*^2

For abbreviation we shall write

$$\phi(U) = \frac{\partial^2 \tau_\omega}{\partial U^2} - \frac{\partial \tau_\omega}{\partial U} \tag{5}$$

Thus we have approximately

$$G \equiv \frac{-\frac{\partial^2 U}{\partial z^2}}{\left(\frac{\partial U}{\partial z}\right)^2} = 1 - \phi(U) \tag{6}$$

when there is no wave induced Reynolds stress, we have uniquely

$$U = \ln z \ , \quad G = 1$$

Solving eq. (6) under the condition that the velocity converges asymptotically to log-law, we have

$$U = \ln (z + H)$$

where

$$H(U) = \int_U^{\infty} e^{U} \left[e^{\int_U^{\infty} \phi\, dU} - 1 \right] dU \tag{7}$$

Thus H corresponds to the zeroplane displacement of log-law. However it is not constant but varies with the height.

The deviation from the log-law on the semi logarithmic scale is given

$$\Delta \ln z = - \ln (1 - H e^{-U})$$

$$H e^{-U} \doteqdot e^{-U} \int_U^{\infty} e^{U} \left(\int_U^{\infty} \phi\, dU \right) dU \tag{8}$$

$$= -2 e^{-U} \int_U^{\infty} e^{U} \frac{\partial \tau_\omega}{\partial U} dU - \tau_\omega$$

2. Wave induced Reynolds stress is given by Miles (MILES, J. 1965)

$$\tau_\omega = \left(\frac{u_1}{u_*} \right)^2 \int_0^{2\pi} \cos^2 \theta\, d\theta \int_{U \cos\theta}^{\infty} \beta \left(\frac{c}{U_1 \cos\theta}, \Omega \right) \Sigma (c, \theta)\, dc \tag{9}$$

where $\Sigma (c, \theta)$ is the slope spectrum of surface waves and β the energy transfer coefficient depending on the wind velocity profile. For logarithmic velocity profile β is numerically computed (MILES, 1959) for adequate profile parameter $\Omega \left(\frac{g z_0}{u_1^2} \right)$

Though the value of β for the wind profile over the sea surface is not known, we shall assume for the sake of simplicity.

$$\beta = 2.3 \left(\frac{c}{u_1} \right)^{1/2} e^{-0.03 \left(\frac{c}{u_1} \right)^2} \qquad for \left(\frac{c}{u_1} \right) \geq 3$$

$$= 0 \qquad for \left(\frac{c}{u_1} \right) < 3 \tag{10}$$

It gives rather small value compared with estimated wind profile over the sea surface, usually approximated by log-law with zeroplane displacement, which gives in general reduced wind profile parameter Ω , perhaps smaller than 3×10^{-3} .

When the power spectrum of ocean waves has been given by

$$S(\sigma)\, d\sigma d\theta = \frac{\alpha}{2\theta_0} g^2 \left(\frac{g}{U_0} \right)^{n-5} \sigma^{-n} e^{-2 \left(\frac{\sigma}{\sigma_0} \right)^{-2}} d\sigma\ d\theta \ (|\theta| < \theta_0) \tag{11}$$

where $\sigma_0 = \frac{g}{U_0}$ (U_0 : wind velocity at the anemometer height),
the slope spectrum is derived at once

$$\Sigma\ (c,\theta)\ dc\theta d\theta = \frac{1}{\theta_0}\frac{(2)^{\frac{n-5}{2}}}{\Gamma\left(\frac{n-5}{2}\right)}\ \overline{s^2}\left(\frac{c}{U_0}\right)^{n-6} e^{-2\left(\frac{c}{U_0}\right)^2}\ d\left(\frac{c}{U_0}\right)\ d\theta \qquad (12)$$

where $\overline{s^2}$ denotes the mean-square slope of the surface.
Then we get the Reynolds stress.

$$\tau_\omega = C_m \int_0^\infty \left(\frac{c}{u_1}\right)^m e^{-b\left(\frac{c}{u_1}\right)^2}\ d\left(\frac{c}{u_1}\right) \qquad (13)$$

where

$m = -\frac{1}{2}$ for $n = 5$ (*Phillipps*)

$m = 0$ for $n = 5.5$

$m = \frac{1}{2}$ for $n = 6$ (*Neumann*)

From previous estimations (MILES. J. 1959, STEWART. R. W. 1961) and empirical results (Munk, 1955) we shall assume

$$\tau_\omega = 0.27 \quad for \quad U = 3$$
$$= 0 \qquad\qquad U < 3$$

Then, assuming $U_0 = 10u_1$ we get approximately

$$C_{-\frac{1}{2}} = 0.340\ ;\ C_0 = 0.200\ ;\ C_{\frac{1}{2}} = 0.105$$

Thus using eq. (8) we can finally calculate the deviation from the logarithmic profile. Fig. 2 shows the computed results compared with observations.

3. Observed value reduced from Takeda's data (TAKEDA 1961) are also shown in Fig. 2. Each observed wind velocities are plotted on semi-logarithmic scale. Then straight line is fitted only to the upper part of each diagram in which we can expect the log-law of velocity distribution. Through these processes friction velocity u_* and roughness length z_0 can be determined. Table 1 contains in addition wind profile parmeter Ω . The reduced observations are limitted only to those for which $\Omega \times 10^2$ is between 2.0 and 4.6 and the differences between surface water temperature and air temperature (about 6 m height) are smaller than 0.7°C, that is, nearly neutral state. Mean wave height during the observations were 10-20 cm and the periods were 2-5 sec.

Table 1. Wind velocity distribution over sea surface (from TAKEDA)

$(U) = \frac{U}{u_1}$ $\qquad$ $(z) = \ln \frac{z}{z_0}$

	u_1(cm)	z_0(cm)	(U_a)	(z_a)	(U_b)	(z_b)	(U_c)	(z_c)	(U_d)	(z_d)	(U_e)	(z_e)	(U_f)	(z_f)	$\frac{gz_0}{u_1^2}\cdot 10^2$
1	74.7	0.11	6.87	6.50	7.16	6.84	7.36	7.17	7.46	7.47	7.86	7.82	8.34	8.47	2.0
2	"	"	6.87	6.37	7.16	6.75	7.26	7 10	7.36	7.46	7.76	7.78	8.33	8.45	"
3	"	"	6.64	6.33	7.10	6.73	7.38	7.09	7.46	7.41	7.84	7.78	8.45	8.45	"
6	"	"	*	*	6.71	6.56	7.18	6.98	7.36	7.33	7.90	7.72	8.42	8.40	"
8	42.6	0.08	6.58	6.11	6.98	6.72	7.16	7.19	7.59	7.58	7.94	8.01	8.69	8.69	4.6
10	74.7	0.16	6.21	5.66	6.52	6.16	6.85	6.59	7.01	6.94	7.50	7.34	8.10	8.02	2.9
12	76.4	0.12	6.09	5.56	6.54	6.23	7.00	6.71	7.13	7.14	7.63	7.56	8.28	8.25	2.0
15	86.0	0.23	5.97	5.58	6.19	5.98	6.40	6.37	6.65	6.67	7.05	7.04	7.67	7.69	3.1
16	80.0	0.20	6.17	5.63	6.46	6.07	6.71	6.45	6.88	6.78	7.35	7.16	7.91	7.82	3.1

4. Our numerical estimate of wind profile rests on the approximated formula of energy transfer coefficient and empirical value of wave induced Reynolds stress in addition to the assumptions of wave power spectrum. Thus the other assumptions could alter the profile considerably.

In spite of these less rational assumptions we conclude that the deviation of wind profile is owing to the reduction of turbulent vorticity transfer (see Appendix).

Appendix

When total shear stress can be regarded as constant and the viscous shear stress is neglected, we can write

$$\tau_e + \tau_\omega = 1 \tag{A.1}$$

From momentum-transfer theory we have well known formula

$$\tau_e = \left(z \frac{\partial U}{\partial z}\right)^2 \tag{A. 2}$$

Thus we get in similar arguments

$$\Delta \ln z \doteqdot \frac{1}{2}\int_U^\infty \tau_\omega \, dU \tag{A. 3}$$

For numerical example, we may assume

$$\tau_\omega = C_1 \int_U^\infty U e^{-0.05U^2} \, dU \tag{A. 4}$$

Assuming

$\tau_\omega = 0.272 \quad for \quad U = 3$

$we\ get\ \Delta \ln z = 0.06 \ , \quad for \quad U = 6$

It gives rather negligibly small deviation.

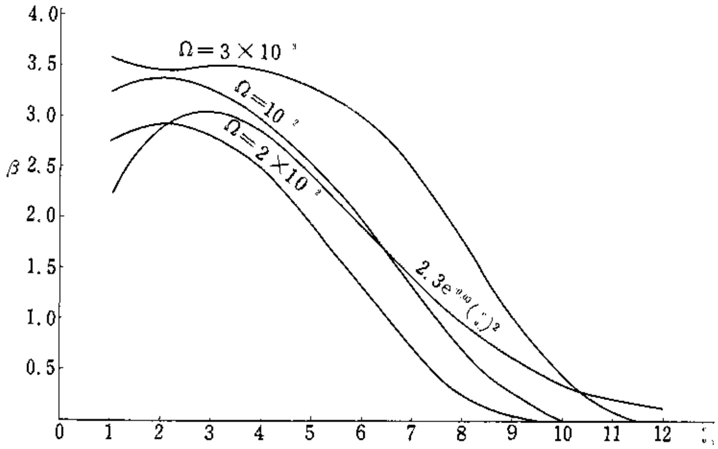

Fig. 1. Assumed value of β compared with ones for logarithmic profile.

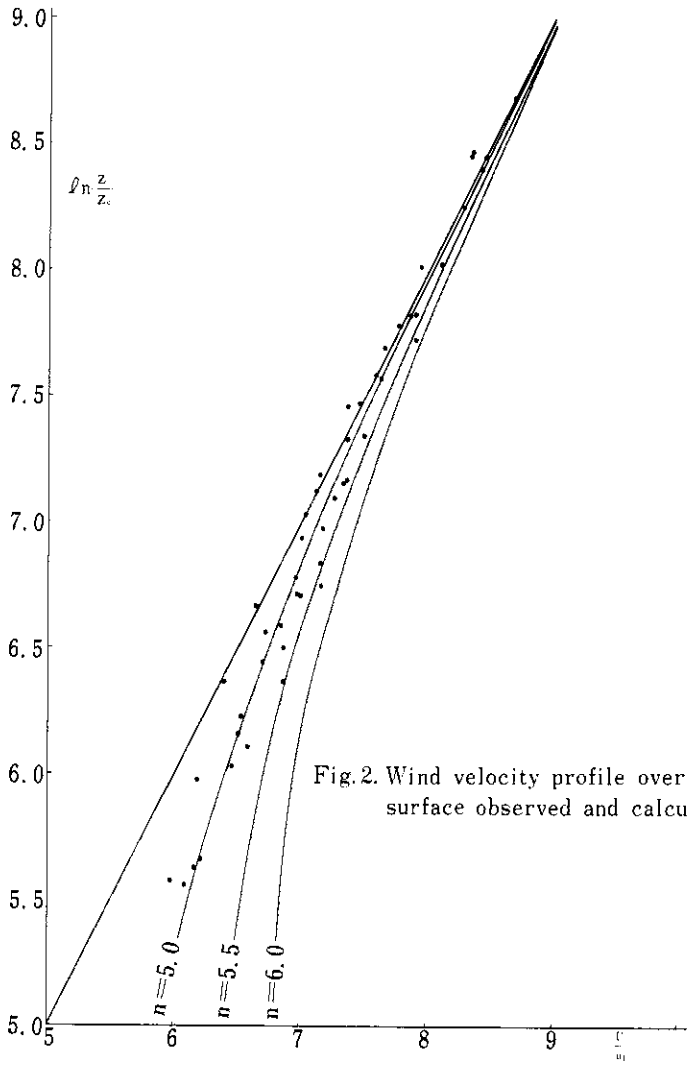

Fig. 2. Wind velocity profile over sea surface observed and calculated.

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