

Theoretical and Morphometrical Analysis of the Erosional Development of Mountain Slopes and Valleys

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Abstract

Evolution of mountain slopes and valleys in temperate and humid regions has been studied by physical consideration of dominant erosive agents, morphometric measurement of erosional landforms mainly of volcanoes and mathematical analysis of theoretical equations.

In a long period of erosion, it is expected that the effects of various erosional factors and phenomena are averaged, and a fundamental process will appear on the surface. Therefore, physical mechanism of erosion can be considered under simple conditions. The dominant erosive agents acting on mountain sides in humid regions are supposed to be the tractive force of flowing water caused by rainfall and the scouring force of moving materials of mass movement.

Assuming that the water flow on mountain sides is in a state of quasi-uniform flow, the following equation *A* of erosion, in terms of topographic factors that can be easily quantified, has been derived by the equation of motion and that of continuity, the formulas for tractional load and the principle of continuity of sediment discharge:

$$E = k_1 l^m \frac{d \sin^n \theta}{d l} + k_2 l^{m-1} \sin^n \theta, \quad (A)$$

where *E* is the erosion rate, *l* the slope length, *θ* the slope angle and *k*₁, *k*₂, *m* and *n* are constants. Also in the case of scouring action of mass movement, the same equation has been derived by assuming the process as the movement of rigid material on the slope. Assuming simply that in a long period of time the removal of the slope forming material is carried on at every portion of the slope in proportion to the magnitude of the erosive force, and introducing the angle of repose for deposition (*θ*_c), the following equation *B* of erosion has been derived by modifying equation *A*:

$$E = K l^{m'} (\sin \theta - \sin \theta_c)^{n'}, \quad (B)$$

where *K*, *m'* and *n'* are constants.

The applicability of equations *A* and *B* to actual processes of erosion has been considered by using the data obtained by morphometric measurements of erosional landforms. The initial slope surface is recovered by burying the valleys and gullies which were made by erosion. By a method of measurement to represent a wide area of a slope in two dimensions, average slope profiles of initial and present landforms are obtained. The distance of the profiles gives the average amount of erosion depth.

For volcanoes ranging in size from a volcanic cone to a large strato-volcano and ranging in erosional stage from middle youth to early maturity, the calculated values of average erosion depths obtained from equation *B* agree quite well with measured ones. The values of *K*, *m'* and *n'* were determined by the method of least squares. Correlation coefficients are more than 0.9. All of them are highly significant. Equation *B* can also be applied with success to the process of slope formation of abandoned

coal slag heaps which were dissected into gullies. At Yotei Volcano the difference in the erodibility due to lithologic state and that in the erosion rate due to slope directions were obtained. The values of m' and n' are nearly 1 for most of measured slopes, irrespective of their erosional stage, location and size. Calculated values obtained from equation A also show agreement with measured values at some of the volcanic slopes. Thus, it has been demonstrated that the erosion of actual mountain slopes is fundamentally carried on through the physical mechanism represented in equations A and B . Equation A has a more reasonable form than equation B , and may have a general applicability. Equation B cannot well describe the depositional phenomena.

The amounts of change in longitudinal profiles of radial valleys are also given by equation B . The reason of applicability of equation B , irrespective of the degree of dissection, can be explained by the assumption that the erosion process at a valley bed is dominant and the recession of valley wall proceeds in proportion to the amount of undercutting of the valley bed while keeping the gradient of the slope almost constant. The observed amounts of the change of valley bed in the Dashibara Valley of the Joganji River show agreement with calculated values from equation A . The dominant process in the valley is that of debris flow. From the results it can be assumed that the fundamental process of erosion is similar, irrespective of the kind of erosive agent.

Many of the processes in which transporting agents do not intervene are represented by independent terms of the gradient or curvature of slope. Most of the mathematical models of slope development proposed thus far have the terms incorporated. However, multiple regression coefficients of these terms are entirely insignificant in the present examples of slopes. These are inferred to be local and insignificant factors of erosion.

By some simplifications and modifications of equations A and B of erosion, the following equations A and B of slope development have been derived:

$$\frac{\partial u}{\partial t} = k_1 x^m \frac{\partial^2 u}{\partial x^2} + k_2 x^{m-1} \frac{\partial u}{\partial x}, \quad (A)$$

$$\frac{\partial u}{\partial t} = K x^{m'} \left(\frac{\partial u}{\partial x} + I_c \right), \quad (B)$$

where u is the elevation, t the time, x the horizontal distance from the divide and I_c a constant. The validity of the equations was confirmed by the agreement of calculated values with measured values of degradation at some slopes.

General processes of slope development have been studied by solving these equations under various initial and boundary conditions. In the early stage of erosion, deposition occurs at the foot of the slope, but in due course of time the re-dissection begins. In a highly dissected stage, a common concave slope which can be approximated with a logarithmic curve is formed independently of the shape of the initial profile. Longitudinal profile of valley bed takes a logarithmic curve in a graded stage. By taking k_1 and k_2 as functions of x near the divide, the formation of convex summit can be shown. Modifying equation A , an equation representing the process in which the divide recedes has been derived. The process of the dissection of a plateau and the development of a radial valley can be represented by this equation. Taking erosional coefficients as functions of location, the changes of slope profiles are considered for the cases where local variations in lithologic condition exist.

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1. INTRODUCTION

Slope is a basic constituent element of the earth's surface, and its form is continually changing under the influence of various erosive agents. Evolution of landforms, especially that of a mountain, is carried on under very complicated conditions in a long geological period. Therefore, it is a difficult but fundamental problem of geomorphology to clarify the mechanism of erosion and the process of slope formation. On account of the complexity of the process and the impossibility of reproducing it on a laboratory scale, it may be considered as difficult and even meaningless to generalize the physical mechanism of erosion process as a whole in quantitative terms. However, as the present landforms we see now have been formed by the cumulative effect of a great number of erosional episodes in a long geological history, supposing a mean and steady condition during the period, it may be possible to consider the erosion process in a simple way with the hope that the effects of various erosional factors and phenomena are averaged, and a fundamental process may appear on the surface.

In the present paper, based on the above idea, the writer tries to derive equations which represent the processes of erosion by theoretical considerations of physical mechanism of dominant processes under simplified conditions, and to confirm the applicability of the equations to the process of erosion on actual mountain slopes by morphometric measurements in order to clarify the mechanism of erosion and the process of slope formation. Here, the term of slope is used in a wide sense, meaning the whole surface of a mountain including valleys.

The amount of erosion in a long period is obtained by an estimation in some way or other. At a slope where the initial slope surface can be estimated, the amount of erosion is obtained by the difference between initial and present slope surfaces. A volcano has suitable conditions; namely, at a volcano that is moderately dissected by valleys and still preserves its initial slope surface, the volume of eroded material at any place of the slope can be fairly correctly estimated by recovering the initial contour lines. Moreover, topographic conditions of it are generally favorable for the treatment of a wide area in two dimensions. Two-dimensional treatment is convenient for the measurement and quantitative analysis of the results. For a similar reason, artificial slopes like coal slag heaps can be taken up for the study.

In the present paper, many strato-volcanoes and coal slag heaps were measured for confirming the applicability of theoretically derived equations. On a general non-volcanic slope, factors related to erosion are generally very complicated and the estimation of the amount of erosion is generally impossible. Therefore, it is hard to confirm the applicability of the equations to a general slope. However, judging from the generality of the consideration tried here, the writer thinks that the erosion process at a general slope is fundamentally the same with that at the slopes considered in the present paper.

Quantitative studies on slope erosion based on observations at actual slopes have been conducted by many researchers. The results, however, should not be generalized directly because there is a possibility that in the short-period process when the amount of erosion is small, the fundamental process does not appear on the surface, owing to the effects of transient and local phenomena. By a practical request of soil conservation, studies on erosion by measurement and experiment have been conducted, and empirical relationships between the amount of erosion and the erosive factors such as rainfall, geology, vegetation, topography and so on have been obtained. Horton

(1945) derived a noteworthy equation of the erosion caused by overland flow, and tried to apply it to soil erosion. Theoretical and experimental studies on the mechanism of erosion have been conducted in the field of hydraulics. Although the results may not be directly extended to actual processes, they give a physical basis for the consideration of erosion process. Mathematical analyses on the process of slope evolution by using mathematical models have been conducted by Culling (1960), Scheidegger (1961), Hirano (1966), Kirkby (1971) and so on. Most of these models, however, have been derived by deductive means and have unsatisfactory points in the fact that they are not quantitatively connected with the dominant erosive agents acting on an actual mountain slope. Therefore, we cannot find any conclusive evidence that a series of curves obtained by solving these mathematical models represents the actual processes of slope development. The mere agreement of natural form and model-derived form is no proof of the validity of the assumption made in constructing the model.

By recovering the initial slope surface and by estimating the amount of erosion, Ruxton and McDougall (1967) and Suzuki (1969) studied the rate of erosion of volcanoes.

Since the slope form evolves through the physical mechanism in a wide sense of various agents, the study of slope development must involve an attempt to quantify and formulate the process. The result must be interpreted in terms of principles of physics and be examined quantitatively in regard to actual landforms.

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2. DERIVATION OF THE EQUATION OF SLOPE EROSION

2.1 Dominant Erosion Processes

The change of a land surface is primarily carried on by the removal of earth material through the action of erosive force. Therefore, the process of erosion may be considered to be equal to the process of transportation when a large change on a land surface is considered. The main erosive agents acting on mountain slopes in temperate and humid regions as Japan are running water and gravity. Wind and groundwater are also acting on mountain slopes as agents. However, the magnitudes are thought to be very small. The rate of weathering is not considered here.

By running water the materials on a slope or in a channel are transported in the forms of tractional load and suspended load. At mountain sides where the gradient is great, the transportation of materials is carried on mainly by tractional action of overland flow and channel flow directly caused by intense rainfall. At the time of no rainfall, running water in the channel has no ability to transport bed material. Therefore, tractive force of running water caused by rainfall can be taken up as a dominant erosive agent.

Erosion process by direct action of gravity is termed mass wasting. This is classified into creep and landslide. Landslide is caused in the forms of slide, flow and fall of earth material. The movement is relatively rapid, and slope form of a mountain is often changed largely by it in a short period of time. Especially, flow and slide entrain the slope forming material during the movement and transport them by traction and suspension. On the other hand, creep is very slow movement of the slope forming material. The greater part of its effect may be eliminated in the actions of

running water and landslides in temperate and humid regions. Thus, scouring force of moving material of mass movement can be taken up as another erosive agent.

2.2 Erosional Mechanism of Tractional Action of Running Water

Part of rainfall is intercepted by the vegetation cover and infiltrates into the ground. The rest of the rain water becomes surface runoff and runs downslope in response to the force of gravity. Sheet erosion takes place by overland flow on the level surface of the slope. Overland flow concentrates gradually into rills and gullies with downhill movement. The depth and velocity of the flow increase, and intense channel erosion is carried on.

Effective rainfall intensity is assumed to be uniform throughout the slope for simplification, though it varies with the elevation and location on the slope. Therefore, the quantity of runoff increases with the distance from the divide. Now, we consider the surface runoff on the mountain slope caused by rainfall to be a uniform sheet flow throughout the slope, assuming a mean and steady condition in a long period of time. Thus, the flow can be treated as two-dimensional. Since the gradient of the slope is great and the change of the quantity of lateral inflow with time is small, it can be assumed that the downslope component of the weight of water is almost equally balanced with the frictional force and the flow is not accelerated downwards, that is, quasi-uniform flow is realized (Ishihara *et al.*, 1962). The equation of motion in this case is shown as follows:

$$-\tau_0 + \rho gh \sin \theta = 0, \quad (2.1)$$

where τ_0 is the frictional resistance, ρ the density of the fluid, g the gravity acceleration, h the depth of the flow, and θ the slope angle. By Manning's formula equation (2.1) is rewritten as

$$\sin \theta - \frac{n^2 u^2}{h^{4/3}} = 0, \quad (2.2)$$

where n is Manning's coefficient of roughness and u the velocity of flow. The equation of continuity of steady flow is represented in the form:

$$\frac{1}{b(l)} \frac{d}{dl} \{ u h b(l) \} = q, \quad (2.3)$$

where $b(l)$ is the channel width, l the distance from the divide and q the effective rainfall intensity. Substituting u obtained from equation (2.2) into equation (2.3),

$$\frac{5 h^{2/3} \sin^{1/2} \theta}{3n} \frac{dh}{dl} + \frac{h^{5/3} \sin^{1/2} \theta}{n} \frac{1}{b(l)} \frac{d\{b(l)\}}{dl} = q \quad (2.4)$$

is derived. When the shape of the slope is fan-like, channel width can be represented as

$$b(l) = al + 1,$$

where a is a constant. Then, by integration under the condition $l=0$, $h=0$, the depth of the flow at a distance l from the divide is given as follows:

$$h_l = \left(\frac{qnL}{\sin^{1/2} \theta} \right)^{3/5}, \quad (2.5)$$

where $L = (l^2 + 2l/a)/2(l+1/a)$. When the channel width is constant, that is, when $d\{b(l)\}/dl = 0$,

$$h_l = \left(\frac{qnl}{\sin^{1/2} \theta} \right)^{3/5} \quad (2.5')$$

is obtained. It is supposed that the change of channel width can be neglected at

general large mountain slopes. The flow entrains and transports the slope forming material by its tractive force. The tractive force τ is expressed as

$$\tau = \rho g h I_e, \quad (2.6)$$

where I_e is the energy slope. In the case of uniform flow, I_e is equal to $\sin \theta$. Hence, from equations (2.5') and (2.6) the tractive force of the flow at l is expressed by

$$\tau_l = \rho g (qn)^{3/5} l^{3/5} \sin^{7/10} \theta. \quad (2.7)$$

Many formulas for tractional load have been proposed thus far. They are expressed in terms of τ and critical tractive force τ_c . At periods of intense runoff when the greater part of erosion occurs, τ is much greater than τ_c . Hence, under the condition of $\tau \gg \tau_c$, the formula for tractional load may be written in the general form:

$$Q_B = c \tau^\alpha, \quad (2.8)$$

where Q_B is the tractional load. By Brown's equation $c = 10/(\sigma/\rho - 1)^2 g^2 d \rho^{5/2}$, $\alpha = 5/2$, and by Sato, Kikukawa and Ashida's equation $c = \varphi F/(\sigma/\rho - 1) g \rho^{3/2}$, $\alpha = 3/2$, where σ is the density of the particle, and d the particle diameter. From equations (2.7) and (2.8), the transport rate of tractional load in weight per unit time and unit width at l is given in terms of the length and the gradient of the slope as follows:

$$Q_{Bl} = K l^m \sin^n \theta, \quad (2.9)$$

where $K = c(\rho g)^\alpha (qn)^{3\alpha/5}$, $m = 3\alpha/5$, $n = 7\alpha/10$. The transport rate at $l + \Delta l$ is

$$Q_{B(l+\Delta l)} = K(l + \Delta l)^m \sin^n \theta_{l+\Delta l}. \quad (2.10)$$

The amount of erosion is given by the increment of Q_B per unit length of the slope, i.e.,

$$\frac{\Delta Q_B}{\Delta l} = \frac{K}{\Delta l} \{ (l + \Delta l)^m \sin^n \theta_{l+\Delta l} - l^m \sin^n \theta_l \}. \quad (2.11)$$

Then, passing to the limit, we arrive at

$$\begin{aligned} E &= \frac{d}{dl} (K l^m \sin^n \theta) \\ &= k_1 l^m \frac{d}{dl} (\sin^n \theta) + k_2 l^{m-1} \sin^n \theta, \end{aligned} \quad (2.12)$$

where E is the erosion or deposition depth measured at a right angle to the slope, $K = k_1$ and $Km = k_2$. Here, equation (2.12) is called equation *A* of erosion. Equation (2.12) is also obtained from equation (2.9) and the relation

$$(1 - \lambda) \frac{\partial y}{\partial t} = - \frac{\partial Q_B}{\partial l}, \quad (2.13)$$

in which $-\partial y/\partial t$ can be replaced with E . Equation (2.13) is the equation of continuity of sediment load.

The coefficient K is a function of the effective rainfall intensity, the roughness coefficient of slope and the density and diameter of particle, that is, a function of erosive factors such as climate, geology, vegetation and topography. The value of K can be supposed to be constant or a simple function of location at each mountain slope, by assuming a mean condition in the long geological history. Thus, relative amount of erosion depth at any point on a slope can be obtained by equation (2.12). Absolute amount cannot be estimated because the value of each factor of K and the duration of erosion cannot be determined even roughly. The values of m and n obtained from Brown's equation are $3/2$ and $7/4$, those from Sato, Kikukawa and Ashida's equation $9/10$ and $21/20$, those from Shields's $8/5$ and $17/11$, and those from

Du Boys's $6/5$ and $7/5$.

In case where the quantity of runoff Q does not increase in proportion to the distance from the divide on account of certain characteristics of the slope or the basin, equation (2.12) can be modified simply by replacing l with Q . This can be confirmed by substituting the relation

$$h = n^{3/5} Q^{3/5} (\sin \theta)^{-3/10}, \quad (2.14)$$

which is derived by transforming Manning's formula into equation (2.9). For the purpose of representing the equation of erosion in terms of topographic factors, it is necessary to express Q by a function of location which represents the form of the basin, local variation in effective rainfall intensity and so on. However, it is difficult to decide the form of the function.

2.3 Erosional Mechanism of Mass Movement

In order to consider the mechanism in a simple way, we assume that the motion of the materials of mass movement can be approximated by the motion of rigid body on a slope. When the frictional resistance is proportional to the second power of the velocity, the equation of motion is given in the form

$$m_s \frac{dv}{dt} = m_s g \sin \theta - k_f v^2, \quad (2.15)$$

where m_s is the mass of moving material, v the velocity and k_f the coefficient of friction. Since the frictional resistance is proportional to the second power of the velocity, the velocity of moving material will soon reach a terminal velocity where the downslope component of the weight of the material is balanced with the frictional force. The terminal velocity is obtained by solving equation (2.15) with $dv/dt=0$, i.e.,

$$v_{\infty} = \sqrt{\frac{g}{k_f} \sin \theta}. \quad (2.16)$$

The frictional force operates as an erosive force. By squaring equation (2.16) it is derived that the erosive force is proportional to $\sin \theta$. Total volume of the material of mass movement increases as it moves down the slope by the entrainment of slope forming material. Therefore, the volume can be expressed in terms of the distance from the divide. Now we assume that the volume passing at l per unit time and per unit width is proportional to l^p (p : a constant) and a mass movement equally occurs at all portion of the slope. Then, the erosive power exerted at l is

$$\begin{aligned} F &= c \int_0^l (l-x)^p \sin \theta dx \\ &= \frac{c}{p+1} l^{p+1} \sin \theta, \end{aligned} \quad (2.17)$$

where c is a proportionality constant. Equation (2.17) has the same form as equation (2.7) which represents the erosive power of running water. Supposing that the transported material is proportional to F and using equation (2.13) for the equation of continuity, equation (2.12) is derived as the equation of erosion of mass movement. Materials of mass movement falling on a long mountain slope do not necessarily reach the bottom of the slope in a continuum, but in an interrupted manner. Therefore, the application of equation (2.12) may be limited to fairly short and steep slopes in the case of mass movement. However, the effect of mass movement falling in a series of intermittent movements may be averaged throughout the slope in a long period of time. Then, we can expect that the erosion is carried on in the way as expressed by equation (2.12).

2.4 Derivation of a Simplified Equation of Erosion

Horton (1945) stated that the total eroding force F_1 at a distance x from the watershed is represented from Du Boys's and Manning's formulas as follows:

$$F_1 = \frac{w_1}{12} \left(\frac{q_s n x}{1020} \right)^{3/5} \frac{\sin \alpha}{\tan^{0.3} \alpha}.$$

Then, assuming that the erosion rate is proportional to the net eroding force, Horton presented the following equation:

$$e_r = \frac{K_e w_1}{12} \left(\frac{q_s n}{1020} \right)^{3/5} \frac{\sin \alpha}{\tan^{0.3} \alpha} (x^{3/5} - x_c^{3/5}),$$

where e_r is the erosion rate, K_e a proportionality factor, w_1 the weight of water, q_s the surface runoff intensity, n the surface roughness factor, α the slope angle, and x_c the critical length of overland flow. Horton thought that the equation is applicable only to uniform overland flow.

Now, we assume after Horton's consideration that the erosion proportional to erosive force occurs at any portion of the slope in a long period of time. Then, from equation (2.7) the following equation is obtained:

$$E = K l^{m'} \sin^{n'} \theta. \quad (2.18)$$

Critical length is neglected since it may be very small compared with the total slope length at the time of heavy rain. The slope angle less than a certain value is needed for the moving material on a slope to come to rest. This is termed the angle of repose for deposition. The value varies with the sort of erosive agent. Takeshita (1963) showed that the angle of repose for deposition by debris flow is 10° — 23° , and that the angle by water flow is less than about 13° . It is observed in many volcanoes that the slope angle where valleys nearly disappear at the base of the slope is fairly great. Then, by introducing a critical angle θ_c , equation (2.18) can be modified as follows:

$$E = K l^{m'} (\sin \theta - \sin \theta_c)^{n'}. \quad (2.19)$$

Here, this is called equation B of erosion. Equation (2.19) can also be derived by using

$$Q_B = c(\tau - \tau_c)^\alpha \quad (2.20)$$

instead of equation (2.8) and by some simplifications. Equation (2.19) means that the force corresponding to the slope angle which exceeds a critical value operates effectively as erosive force. Though equation (2.19) is not fully theoretical and cannot represent the deposition phenomena, it may be applied to slopes which have favorable conditions.

3. APPLICATION OF THE EQUATIONS TO ACTUAL PROCESSES

3.1 Measured Slopes and the Method of Measurement

Since equations A and B of erosion are derived theoretically under simplified conditions, the validity cannot be accepted unless they are interpreted in regard to actual landforms and field relationships. Then, morphometric measurements of moderately dissected strato-volcanoes and coal slag heaps in Japan have been performed to confirm the applicability to actual processes of erosion.

Measured slopes were chosen under the following conditions: the recovery of initial landform can be performed objectively, landform deformation by the causes other than the direct action of erosive force to the slope is supposed to be little, the form of the mountain is conical or the slope surface is flat, the base of the slope is smooth with gradually decreasing slope angles, the form of the slope surface and the

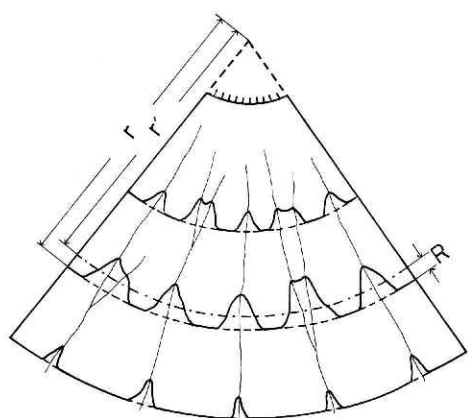


Fig. 1. Method of measurement (1). R : average amount of slope recession; dash-line: recovered initial contour line.

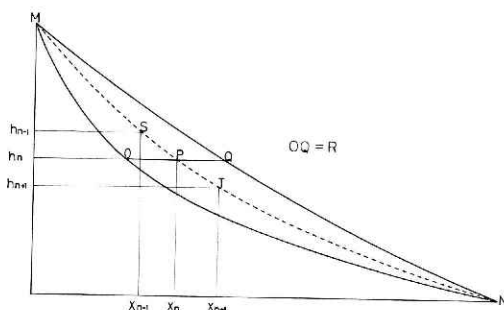


Fig. 2. Method of measurement (2).
MQN: initial profile; MON: present profile.

channel of valleys are not so deformed by lateral volcanoes and adjacent mountains, and the shape of drainage basin is fan-like or rectangular.

The method of measurement is as follows. At first, both sides of the slope are drawn with straight lines, taking account of the arrangement of valleys. The initial contour lines are recovered by smoothly connecting the contour lines which represent the initial surface over valleys and gullies formed by erosion. The method of the recovery of the initial slope is shown in Fig. 1. The embayment of contour line by the distortion of initial slope can be excluded since it is unnatural in the case of a valley. Then, the fan-shaped or rectangular areas enclosed by each of the contour lines of initial and present land surfaces, both sides and the divide are measured. The mean distance from the divide to the place with its altitude the same as that of the measured contour line is given by the radius of the fan or by the length of the longitudinal side of the rectangle of which areas are the same as that of the measured area. Taking the altitude on the ordinate and the distance on the abscissa, average slope profiles of initial and present slope surfaces are obtained (MQN and MON in Fig. 2). In Fig. 2, OQ represents the average amount of slope recession R at the altitude. Since erosive action is directed normal to the slope, the amount of slope deformation measured normal to the slope is obtained and compared with the magnitude of erosive force. As shown in Fig. 5, at the slope which is youthfully dissected, the distance between the initial and present average profiles is small. Then, assuming that the slope profile MPN in Fig. 2, obtained by connecting middle points of OQ, represents the mean condition during the erosion period, the topographic factors such as slope angle and slope length are measured by the profile. Gradient of slope is calculated by the relation $\tan \theta = -(h_{n+1} - h_{n-1}) / (x_{n+1} - x_{n-1})$. Average amount of erosion depth E_m is obtained by $E_m = R \sin \theta$, when the slope degradation is small.

Major premises for the measurement are as follows: that the slopes taken up were formed into smooth surfaces in a short period of time, and that the valleys which had been formed before the formation of the present slopes were completely buried beneath the surface, and hence, that the initial land surface can be obtained by burying valleys and gullies. These premises may be fully realistic, if the slope is reasonably selected and the measured part is aptly limited.



Fig. 3. Location map of measured slopes.

By this method of measurement the sheet erosion and the rill erosion are excluded because they do not form any valleys large enough to be represented in topographic maps. However, since it is generally observed that mature soil develops and the layer of volcanic ash is well preserved on mountain sides, the surface layer of mountain sides excepting that of valley slopes is supposed to be nearly immune from erosion. Therefore, sheet erosion is thought to be small in quantity.

The scale of topographic map used here ranges from 1/5,000 to 1/50,000. Maps of 1/50,000 were used where the size of the mountain was large and the slope surface was highly dissected. The area was measured by using a dot templet. Since the

accuracy of measurement can be improved by increasing the number of measurement, the accuracy of morphometry as a whole is determined mainly by the accuracy of topographic maps. At steep upper parts of slopes, the embayments of contour lines which represent valleys are relatively small. Therefore, the accuracy of measurement may be low at the upper part of the slope. However, at most parts of the slope, satisfactory accuracy is thought to be obtained.

3.2 Application of Equation B

3.2.1 Method of Calculation

At first, we try to apply equation B of erosion which has a simpler form for the actual process of erosion. The age of the slope is generally unknown, and it is very difficult to determine the absolute value of each factor of K which varies with time and location. So it is almost impossible to obtain the absolute amount of erosion by the equation. If possible, intentional management is inevitable. If equations A and B represent actual processes, the values calculated from these equations, of which the variables are topographic factors objectively obtained, should show a relative agreement in their local variations with measured values. The values of m' and n' are thought to vary with the characters of the slopes and with erosional phenomena which occur there. Therefore, we confirm the applicability of the equation by the existence of significant correlation between the measured values and the calculated values obtained from determining unknown coefficients by the method of least squares.

Equation (2.19) can be made linear with regard to unknown quantities by taking the logarithms of both sides of the equation as follows:

$$\log E = \log K + m' \log l + n' \log (\sin \theta - \sin \theta_c). \quad (3.1)$$

Multiple regression coefficients $\log K$, m' and n' can be obtained by the method of least squares. The values of E_i , l_i and θ_i are obtained by measurement so many as the number of measured contour lines. The angle of the lower part of the slope where the amount of erosion becomes nearly zero was given for θ_c . The places, where the accuracy of measurement was supposed to be low and where the erosion proceeds abnormally for a certain reason, were excluded.

3.2.2 Examples of Application

- (1) Little dissected volcanoes. The equation of erosion was derived by assuming

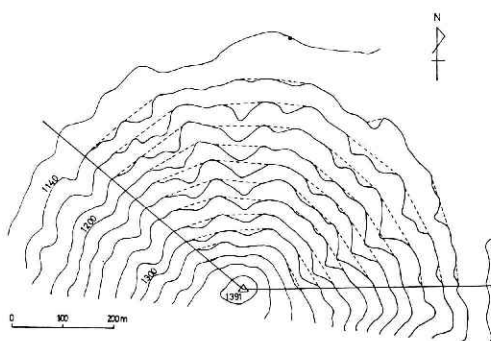


Fig. 4.1 Measured part of Mt. Harunafuji.
Dash-line: recovered initial contour line.

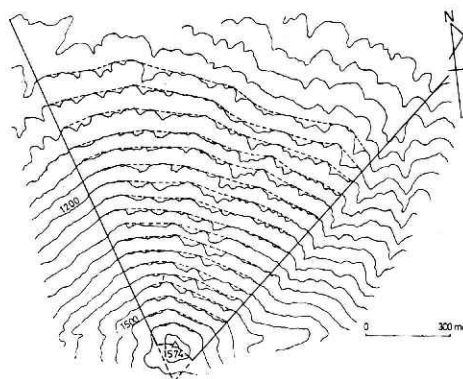


Fig. 4.3 Measured part of Mt. Takachiho.

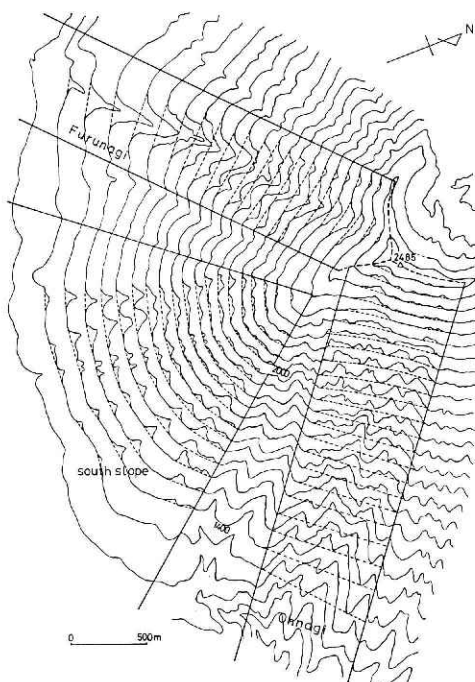


Fig. 4.2 Measured parts of Mt. Nantai.

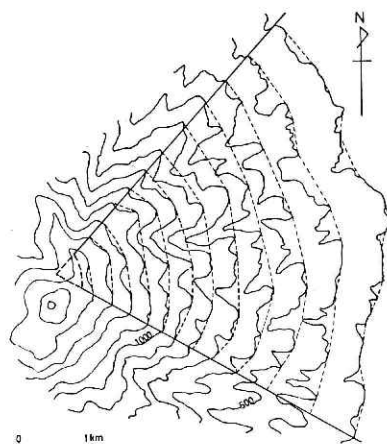


Fig. 4.4 Measured part of Mt. Iwaki.

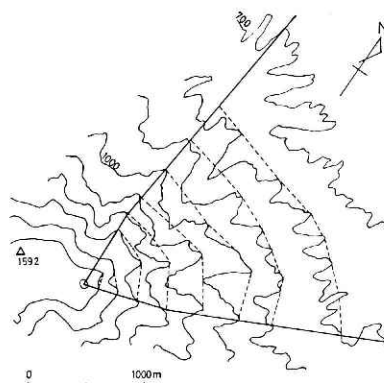


Fig. 4.5 Measured part of Mt. Takadake.

uniform sheet erosion. Therefore, in the first place, the application is limited to the slopes where valleys are not so large. Mt. Harunafuji, Mt. Nantai, and Mt. Takachiho were taken up here. Maps on a scale of 1/5,000 prepared by aerial surveying were used. The measured parts and recovered initial landforms are shown

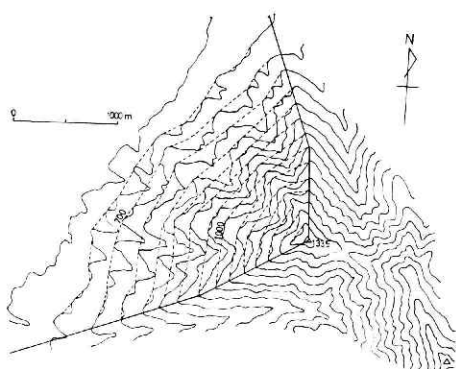


Fig. 4.6 Measured part of Mt. Maefurano.

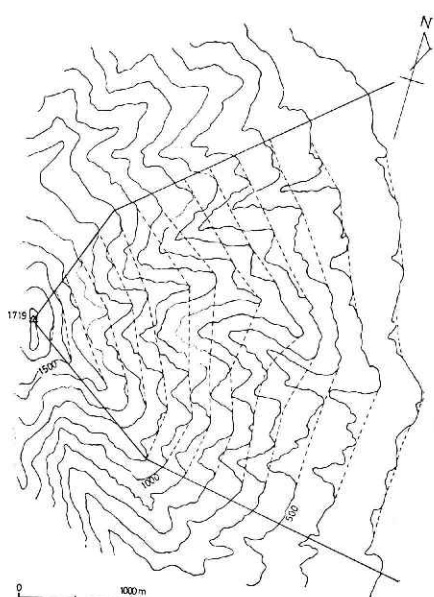


Fig. 4.8 Measured part of Mt. Rishiri.

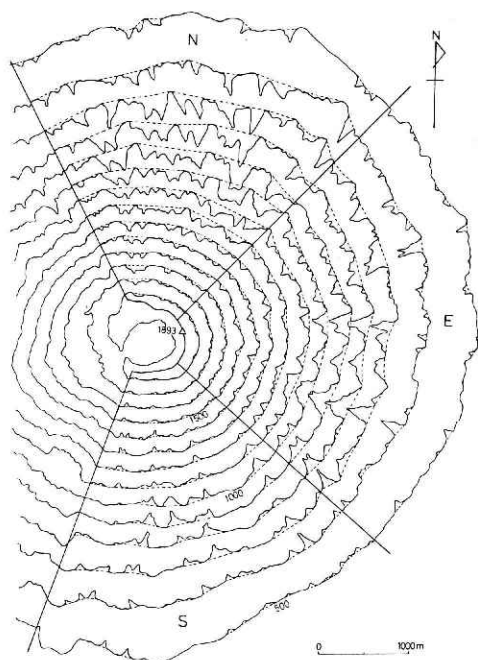


Fig. 4.7 Measured parts of Mt. Yotei.

in Fig. 4, together with other slopes which are to be considered later. Mt. Harunafuji is a small volcanic cone which was formed within the caldera of Mt. Haruna. The measured part was restricted to the northward-facing slope where valleys were large enough to be measured. At Mt. Nantai the south slope whose shape is conical was taken up. The measured part

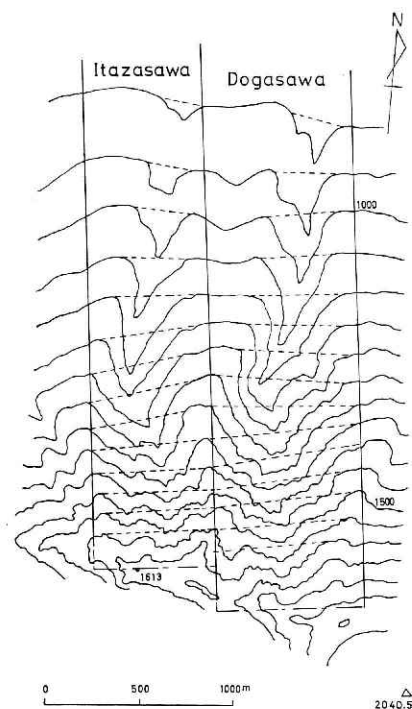


Fig. 4.9 Measured parts of Mt. Iwate.

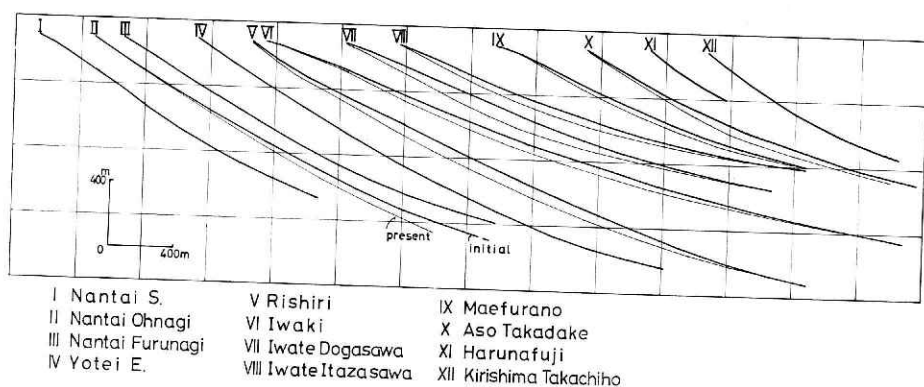


Fig. 5. Average profiles of initial and present landforms.

of Mt. Takachiho is the north slope where the shape of the slope and the arrangement of valleys are favorable; the amount of erosion depth does not decrease because of intense headward erosion of a large valley lying at the base of the slope. The intervals of measured contour lines are 20 m at Harunafuji, 50 m at Nantai and 50 m at Takachiho.

Average profiles of initial and present slopes are shown in Fig. 5 together with other slopes to be considered later. In Fig. 6 measured amounts of erosion depth are compared with calculated ones (in m). Elevation is taken on the abscissa since measured values are obtained for each of contour lines. Correlation coefficients of calculated values with measured ones are 0.994 for Mt. Harunafuji, 0.988 for Mt. Nantai and 0.946 for Mt. Takachiho. All of the correlation coefficients are significant at the 0.001 level of significance. Nearly complete correlations are obtained for the mountains Harunafuji and Nantai which have favorable conditions. Thus, from the results, it is confirmed that at these slopes there proceeds the erosion under the physical mechanism represented in equation *B* in the mean condition during a long period of time.

(2) Well dissected volcanoes. Next we try to apply equation *B* to volcanic slopes in the stages of late youth and early maturity. Measured volcanoes are Mt. Iwaki, Mt. Rishiri, Mt. Maefurano, and Mt. Takadake. The scales of maps used and measured contour intervals are respectively 1/25,000 and 50 m for Mt. Maefurano, and 1/50,000 and 100 m for other volcanoes.

In Fig. 7, comparisons of the measured amounts of average erosion depth with the calculated values are shown. For every slope, both of the measured and calculated values agree well with each other. Correlation coefficients are 0.990 for Mt. Iwaki, 0.960 for Mt. Rishiri, 0.986 for Mt. Maefurano and 0.931 for Mt. Takadake. The correlations are highly significant. Thus, it is shown that the erosional mechanism represented in equation *B* is also acting in fairly highly dissected volcanoes of which initial surfaces begin to disappear on the upper slopes and in large strato-volcanoes whose slope lengths are more than 4 km. It is difficult to confirm the applicability of the equation for the highly dissected mountains where greater parts of the initial surfaces have disappeared because the initial surfaces cannot be recovered accurately by morphometry. However, judging from the general characteristics of the mechanism represented in the equation, it may be applied to maturely dissected slopes. The depths of valleys become more than 100 m at volcanoes of early mature stages.

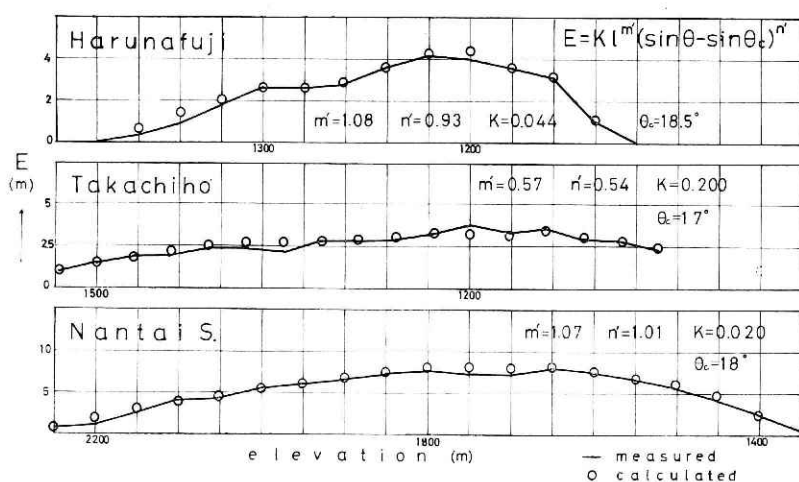


Fig. 6. Comparison of measured amount of erosion depth with the calculated. Little dissected volcanoes.

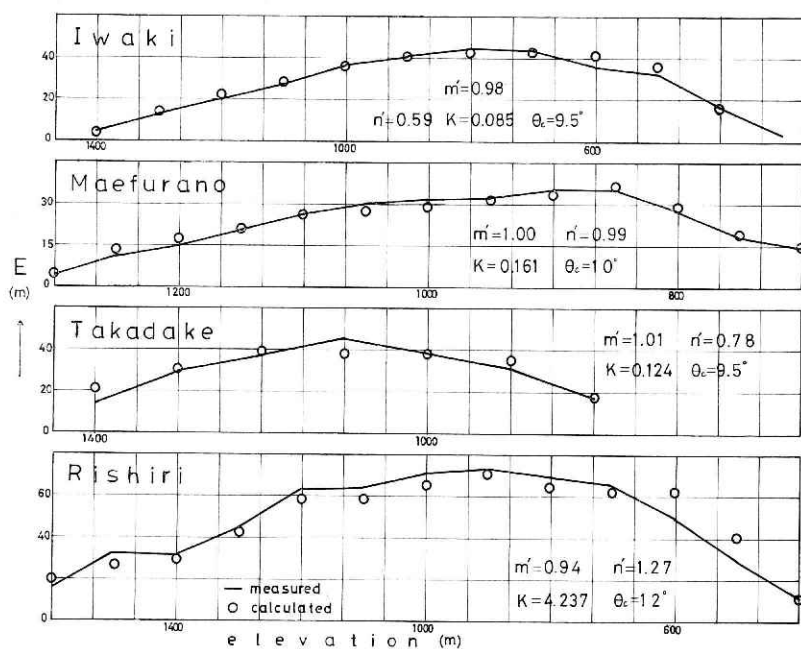


Fig. 7. Comparison of measured amount of erosion depth with the calculated. Well dissected volcanoes.

Nevertheless, equation B gives a good approximate value of erosion depth. Therefore, the application of equation B may not be limited to the processes which can be approximated by sheet erosion.

(3) A drainage area. The shape of slopes considered so far is fan-like or conic. In such slopes the increase of discharge with the increase of l is represented by equation (2.5), and the measured amount of average erosion depth becomes smaller at the lower

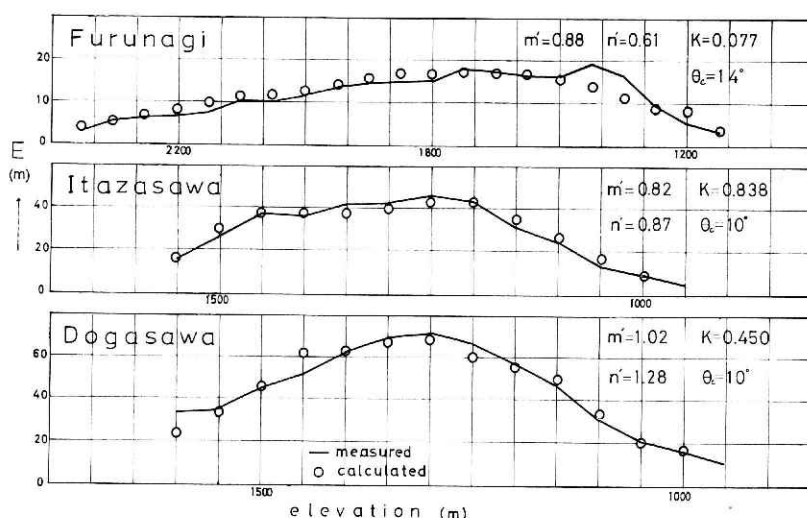


Fig. 8. Comparison of measured amount of erosion depth with the calculated.
A drainage area.

part of the slope where the width of the slope is large and the greater part of the initial surface is preserved. And erosion is carried on, being closely related to the characteristics of drainage basins. Therefore, we try to apply equation B to volcanic slopes which have a radial valley. Measured areas were so limited as to make rectangular shapes.

Radial valleys under consideration are Furunagi valley at the southeast part of Mt. Nantai, and Dogasawa and Itazasawa valleys on the north slope of Mt. Iwate. At Mt. Iwate, imaginary divides were drawn at appropriate locations by taking the shape of the mountain into account. The map of Mt. Iwate was made from aerial photographs by using a stereo-micrometer on a scale of $1/20,000$.

Comparisons of measured values with calculated ones are shown in Fig. 8. Correlation coefficients are 0.974 for Furunagi V., 0.965 for Dogasawa V. and 0.977 for Itazasawa V. The correlations are highly significant. At Furunagi valley the data around the altitude of 1,400 m were excluded, since the erosion proceeded abnormally there, due to the collapse of sheet lava. Although the valley walls of the upper part of the valleys Dogasawa and Itazasawa are very rugged, good agreements are obtained. From the result it may be concluded that the applicability of equation B is independent of the shape of the slope and the number of valleys.

(4) Landslide. Ohnagi valley in the southeast part of Mt. Nantai is a valley where landslide is the main process. Since landslide is supposed to be a very unsteady phenomenon, theoretical treatment of the process may be generally difficult. As the initial landform of Ohnagi valley can be estimated by the way shown above, we try to apply equation B to it. Both the slope sides were drawn with parallel straight lines to form a rectangular slope. The lower part of the slope could not be measured because the landform was complicated there.

Making $\theta_c=0$ and $n'=0.5$, the relation $E=0.0008 l^{1.51} (\sin \theta)^{0.50}$ was obtained. As shown in Fig. 9, the measured values agree well with the calculated. The correlation coefficient is 0.992. By the relation $E=0.0012 l^{1.40}$ which is obtained by neglecting the factor of $\sin \theta$, the calculated values also show a good agreement with the

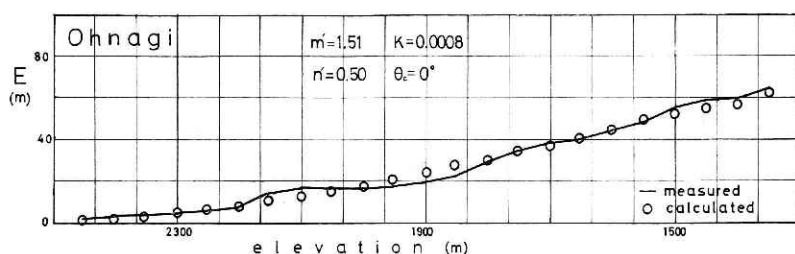


Fig. 9. Comparison of measured amount of erosion depth with the calculated. Landslide.

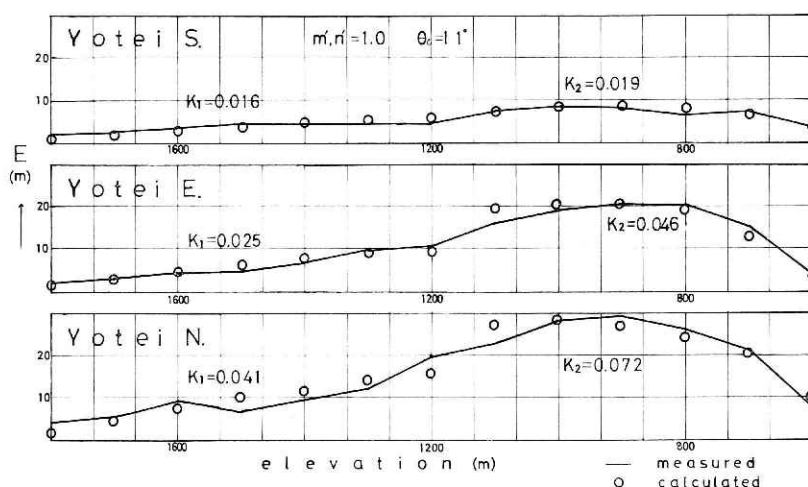


Fig. 10. Comparison of measured amount of erosion depth with the calculated. Mt. Yotei.

measured. From the result it may be assumed that the erosion process of landslide can be represented by equation B.

(5) Mt. Yotei. Yotei strato-volcano is worthy to be considered because its shape is typically conic with its smooth foot and uniformly developing radial valleys. As to this volcano, the difference in the values of coefficient K with different slope directions and lithologic conditions can be estimated. As shown in Fig. 4.7, the slope was divided into three parts in order to be measured respectively apart. The west slope was excluded since it was covered with the latest lava flow and there were no valleys well-developed. Measured slopes are thought to be formed at the same time. A map on a scale of 1/25,000 was used for the measurement. The upper slope situated above the altitudes of 1,100–1,300 m is covered with lava, and the lower slope with pyroclastic material. Due to the difference in the lithologic condition, the sizes of valleys change discontinuously at the said altitudes. Timber line is also situated around the altitudes. In Fig. 10, altitudinal changes of average erosion depths are shown. In every slope the amount of erosion depth changes at these altitudes. So, the values of K were calculated at the upper and lower slopes, respectively. The boundary was set at the altitude of 1,150 m. In order to avoid the effect of the

differences in m' , n' and θ_c , calculation was performed by making $m'=n'=1$ and $\theta_c=11^\circ$.

The equations in a general form used in the calculation are $E_1=K_1l(\sin\theta-\sin 11^\circ)$ for the upper slope and $E_2=K_2l(\sin\theta-\sin 11^\circ)$ for the lower slope. The values of K_1 and K_2 are 0.041 and 0.072 at the north slope, 0.025 and 0.046 at the east and 0.016 and 0.019 at the south. Correlation coefficients of calculated values with measured ones are 0.958 at the north slope, 0.980 at the east, and 0.946 at the south. They are highly significant. As the boundary line between the lava slope and the pyroclastic slope is not straight but rugged in reality, the differences between the measured and calculated values are fairly large at the portions of 1,100 m and 1,200 m.

The coefficient K represents the erosion rate or the erodibility of the slope. The ratios K_2/K_1 are 1.76 at the north slope, 1.84 at the east, and 1.19 at the south. Excluding the value of the ratio at the south slope where the lithologic condition may be unfavorable, 1.8 is obtained for the value of K_2/K_1 . This means that the erodibility of pyroclastic slope is 1.8 times as large as that of the lava slope at Mt. Yotei.

Table 1. Physiographical factors and

	maximum altitude	relief	slope length	average gradient
	m	m	m	
Harunafuji	1380	250	490	30°55'
Nantai, S.	2300	950	2050	27 50
Takachiho	1560	(570)	(1170)	29 10
Iwaki	1430	1080	3800	16 30
Rishiri	1720	1370	3500	23 10
Maefurano	1330	655	1900	20 10
Takadake	1590	790	2170	21 35
Furunagi	2450	1100	2590	25 35
Dogasawa	1720	745	2590	17 00
Itazasawa	1615	665	2280	17 10
Ohnagi	2480	(1150)	(3200)	27 35
Yotei, S.	1870	1320	3100	25 30
Yotei, E.	1890	1340	3200	24 55
Yotei, N.	1855	1360	3500	23 05
Mitsubishi	108	61	113	33 15
Daishojin	117	57	110	30 10

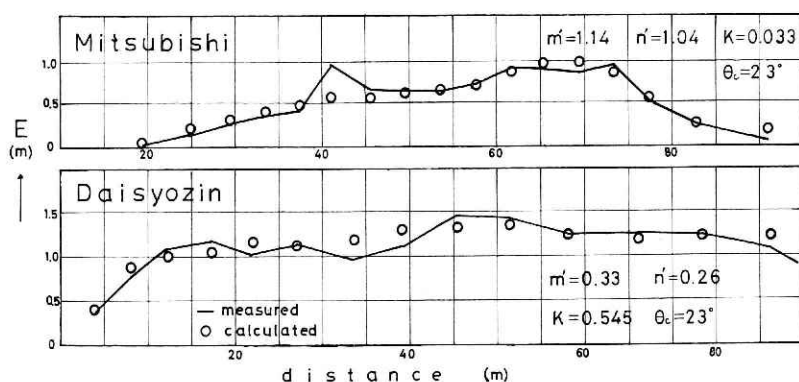


Fig. 11. Comparison of measured amount of erosion depth with the calculated. Coal slag heap.

coefficients of measured slopes.

rock type	m'	n'	θ_c	E_{\max}	\bar{R}/r
				m	$\times 10^{-2}$
dacite	1.08	0.93	18.5°	4.2	1.27
pyroxene andesite	1.07	1.01	17	8.1	0.51
ditto	0.57	0.54	18	3.9	(0.55)
ditto	0.98	0.59	9.5	46.0	2.68
basalt, andesite	0.94	1.27	12	73.0	4.24
ditto	1.00	0.99	10	36.0	3.60
ditto	1.01	0.78	9.5	45.5	4.64
pyroxene andesite	0.88	0.61	14	19.9	1.20
basalt, andesite	1.02	1.28	10	71.5	5.14
ditto	0.82	0.87	10	46.7	4.04
pyroxene andesite	1.51	0.50	—	(66.0)	—
ditto	(1.00)	(1.00)	11	9.0	0.71
ditto	(1.00)	(1.00)	11	21.0	1.56
ditto	(1.00)	(1.00)	11	29.5	1.83
coal slag	1.14	1.04	23	0.94	1.13
ditto	0.33	0.26	23	1.47	1.28

The effects of the differences in vegetative and climatic conditions may be contained in this value, but they are thought to be small compared with the effect of lithological condition in this case. As the measured parts of the mountain are thought to have been formed at the same time, the difference of K indicates the difference of erosion rate in the different directions of the slope. The value of K_1 relative to the value 1.00 at the north slope is 0.61 at the east slope and 0.39 at the south slope, and the value of K_2 relative to the value 1.00 at the north slope is 0.64 at the east slope and 0.26 at the south slope. From the result the erosion rate of the east slope is inferred to be 0.6 times as much as that of the north slope, and that of the south slope is inferred to be 0.3 times. The intensity of erosion differs with the direction of the slope on account of the difference of climatic conditions such as precipitation, insolation and wind direction. At the slope facing north, the effective rainfall intensity is generally greater and erosion proceeds faster.

(6) Coal slag heap. Some of abandoned coal slag heaps are dissected into gullies as if into radial valleys. Coal slag heaps in general may not be included in non-volcanic slopes, since they are similar to volcanoes in respect to their forming process and composing materials, but now, we try to apply equation B to some of them as an example of general slopes.

The coal slag heaps considered are the Mitsubishi Iizuka heap and the Daishojin heap to the south of Iizuka City in Kyusyu. The relative height of Mitsubishi coal slag heap is 60 m, and its mean slope angle is 33° . Mean maximum depth of the gullies is 2—3 m. There exists a belt of no erosion of 20 m in width around the summit. Daishojin coal slag heap is 60 m in relative height and 30° in mean slope angle. The heap is fairly dissected and a few wide and deep gullies are formed by abstraction. The materials of both the heaps are unconsolidated particles of coal, coal ash and rock fragments.

In Mitsubishi heap, a gully at the northwest slope was surveyed by using a hand level and a measuring tape. Cross-sectional areas of the gully were surveyed at 3-m intervals in height. As the gullies were represented as sufficiently large enough to be measured in the topographic map of Daishojin on a scale of 1/3,000, morphometric measurement was performed at Daishojin to take up an area as wide as possible though the accuracy might be low.

In Fig. 11, comparisons of measured values with calculated ones are shown. Correlation coefficients are 0.994 for Mitsubishi heap and 0.726 for Daishojin heap. The correlation coefficients are highly significant. Thus, the applicability of equation B to the erosion of coal slag heap has been confirmed, though the duration of erosion is only a few years. The value of θ_c is great, probably because the effective runoff intensity is small due to the small size of the slope and to its high permeability.

3.2.3 Consideration of the Result

As shown above, morphometric measurements have been performed at various kinds of volcanoes which cover a wide range of conditions, from large strato-volcano in early mature stage, like Mt. Rishiri, to little dissected small volcanic cone, like Mt. Harunafuji, and at coal slag heaps dissected by gullies. Calculated values of average erosion depth obtained from equation B of erosion with determined coefficients show good agreements with measured values at all of these volcanoes and coal slag heaps. Thus, it may be clarified that the erosion of actual mountain slopes proceeds through the physical mechanism represented in equation B . Since most of valleys of the considered slopes are ephemeral because of high permeability, the erosion process by mass movement is supposed to take a considerably large part. Therefore,

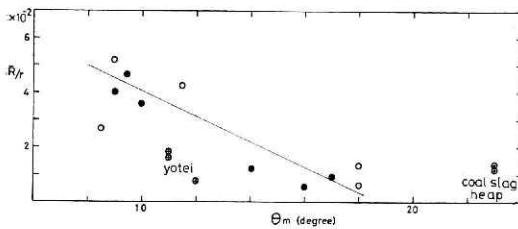


Fig. 12. Relation of the angle of the lower end of the slope (θ_m) to the degree of dissection (\bar{R}/r).

the same in every case. The values obtained from formulas for tractional load are in a range from 0.9 to 1.75. The value of m' is nearly equal with that of n' at each slope. Therefore, equation (2.19) can be modified as follows:

$$E = K(l \sin \theta - l \sin \theta_c)^p. \quad (3.2)$$

As $l \sin \theta_c$ may correspond to the critical tractive force, physical meaning of the above equation is clearer. The values of m' and n' are supposed to be related to the kinds and aspects of occurrence of the erosional phenomena which take place there and to the local variations of erosional factors such as infiltration capacity, vegetation cover, and lithology. The coefficient K may be termed erosional coefficient. Erosion rate can be assumed by dividing K with the period of erosion.

It is generally observed at volcanoes that the outlets of larger valleys are situated at gentler portions of the foot of the mountain. This may be caused by the fact that the larger the valley is, that is, the larger the drainage area is, the gentler is the gradient of the bed where the tractive force of flowing water decreases down to a critical value. In Fig. 12, the relation of the angle of the lower end of the slope θ_m to the degree of dissection D_d is shown. The value θ_m is the angle of the average profile where average erosion depth comes to nearly zero. In some volcanoes θ_m is slightly different from θ_c . The degree of dissection was calculated with \bar{R}/r , where \bar{R} is the average amount of slope recession and r the horizontal distance of the slope. Figure 12 shows that as erosion proceeds θ_m decreases, that is, the outlet of the valley is extended to a gentler part. The values for volcanoes of which r 's are about 2 km are shown with black circles in Fig. 12. In this case the correlation is clearer because the effect of the size of the slope is eliminated. Thus, θ_m is thought to be a function of erosional stage. There is a certain upper limit for the increase of drainage area because the size of the mountain is finite and the neighboring valleys exist. Hence, θ_m may have a lower critical value. By Fig. 12, the value of about 9° is suggested for the critical value. The values of coal slag heaps are very much apart from a regression line since the slopes are very small.

3.3 Application of Equation A

Here, we consider the applicability of equation A of erosion to actual processes. The values of m and n of equation A cannot easily be obtained by the method of least squares since the equation is non-linear with regard to unknown coefficients. Then, we take $n=1$ for simplification. Thus, $d \sin \theta / dl$ corresponds to the curvature of slope. In order to determine the value of m , calculations were performed by giving various values for m . Calculated values agree quite well with measured ones when the values of m are set at 1.8–2.0. This means that the value of α in equation (2.8) is about 3 in the case of steep slope. Then, we take $m=2$. The value of $d \sin \theta / dl$

it is assumed that the fundamental process of erosion is not altered by the duration of erosion and the sort of erosive agent.

In Table 1, physiographic factors and coefficients of measured slopes are shown. The values of m' and n' are about 1 independently of the location, size and erosional stage of the slope. This means that the dominant erosive agent is almost

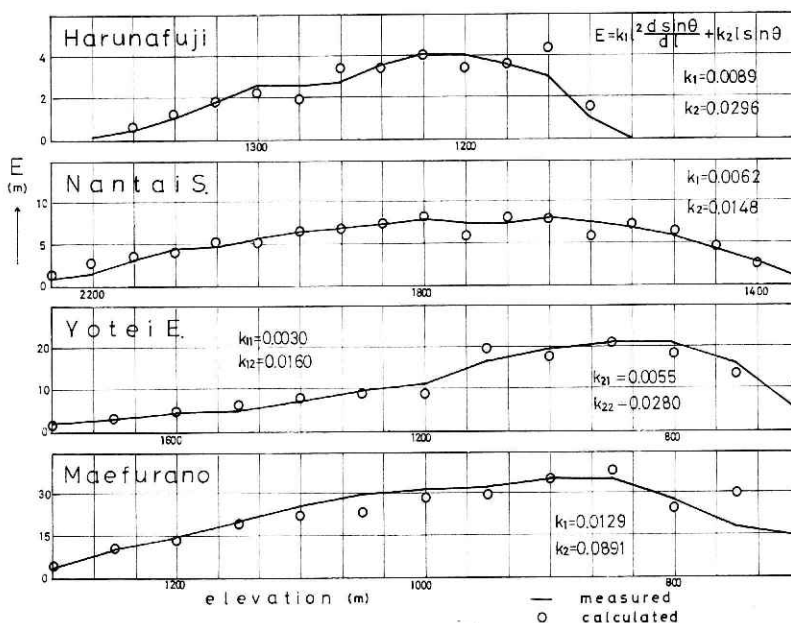


Fig. 13. Comparison of measured amount of erosion depth with the calculated. By equation A.

was calculated by $-2\{(h_{n+1}-h_n)/PT-(h_n-h_{n-1})/SP\}/ST$. The curvature of slope is changed considerably even by slight local distortions of the slope and cannot be accurately measured by the simplified measurement. Therefore, we try to apply equation A to the slopes whose shapes are fairly uniform. The south slope of Mt. Nantai, the slopes of Mts. Harunafuji and Maefurano and the east slope of Mt. Yotei were chosen from among the slopes previously measured.

Comparisons of measured values with calculated ones are shown in Fig. 13. The coefficients of the first and second terms of the equation used for calculation were obtained by the method of least squares. Calculated values fluctuate considerably in the cases where the distortions of average slope profiles are fairly large. Nevertheless, both the values agree well with each other at every slope. Correlation coefficients of calculated values with measured ones are 0.934 for Mt. Harunafuji, 0.941 for Mt. Nantai, 0.970 for Mt. Maefurano and 0.937 for Mt. Yotei. All of the correlation coefficients are highly significant. Thus, it is confirmed that equation A represents the erosional mechanism acting on actual mountain slopes here measured. Equation A has a more general form compared with equation B, since equation A is thoroughly theoretical and its physical meaning is clear. The first term of the right-hand side of equation A gives the amount of deposition at the concave parts of slopes. Therefore, it can well represent the disappearance of valleys and the transition from an erosional to a depositional area at the base of the mountain.

In the previous consideration, topographic factors were measured by the average slope profile shown by MPN in Fig. 2. As the curvature of slope which is affected sensitively by local distortions of slope profile is concerned in the case of equation A, we try to calculate by the calculus of finite differences instead of the simple method. Rewriting equation (2.12) by using the amount of slope recession R and taking $m=2$

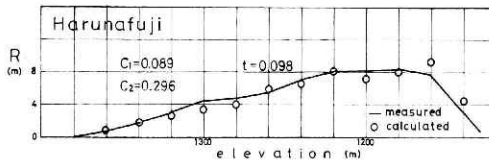


Fig. 14. Comparison of measured amount of slope recession with the calculated. Calculus of finite differences.

and $n=1$, the following relation is obtained:

$$\begin{aligned} \frac{\Delta R}{\Delta t} &= \frac{\Delta E}{\Delta t} \times \frac{1}{\sin \theta} \\ &= c_1 l^2 \frac{d \sin \theta}{dl} \frac{1}{\sin \theta} + c_2 l. \quad (3.3) \end{aligned}$$

By using the difference equation derived from equation (3.3), the slope profiles in the succeeding steps of time can be ob-

tained, starting from an initial profile.

In Fig. 14, the result of calculation of Mt. Harunafuji is shown. The calculations were performed by taking $\Delta t=0.0001$, $c_1=0.089$ and $c_2=0.296$ which are the values previously obtained. Calculated value of R shown in Fig. 14 is that at $t=0.098$. Better agreement is obtained compared with the result shown in Fig. 13. The correlation coefficient is 0.950. If m is determined by the method of least squares, much better agreement will be obtained. As it is generally considered that the curvature of slope is a regulating factor of erosion, it may be reasonable to take $n=1$.

4. CHANGE IN LONGITUDINAL VALLEY PROFILE

4.1 Erosion Process in Valley Bed

We consider the process of erosion in valley bed where erosion is carried on intensely. The dominant erosive agents in valley bed are the tractive force of flowing water caused by heavy rain and the scouring force of mass movement in the form of flow.

As the gradient of valley bed is fairly great, the flow can be approximated by uniform flow. Then, the equation of motion is given by

$$g \sin \theta - \frac{\tau_0}{\rho R} = 0. \quad (4.1)$$

By using Manning's formula equation (4.1) is rewritten as

$$\sin \theta - \frac{n^2 u^2}{R^{4/3}} = 0, \quad (4.2)$$

where R is hydraulic radius. The equation of continuity is

$$\frac{dQ}{dl} = q', \quad (4.3)$$

where Q is the discharge, and q' the lateral inflow by rainfall and from valley walls. Assuming that the cross section of the channel is V-shaped, the relation between cross-sectional area A and hydraulic radius is represented in the form:

$$A = cR^2, \quad (4.4)$$

where c is a proportional coefficient. As discharge is the product of cross-sectional area and velocity, equation (4.3) is rewritten as

$$\frac{d(cR^2 u)}{dl} = q'. \quad (4.5)$$

From equations (4.2) and (4.5),

$$\frac{8c R^{5/3} \sin^{1/2} \theta}{3n} \frac{dR}{dl} = q' \quad (4.6)$$

is derived. Solving equation (4.6), hydraulic radius at the distance l from the divide is given as follows:

$$R = \left(\frac{q'nl}{c \sin^{1/2} \theta} \right)^{3/8}. \quad (4.7)$$

Then, the tractive force of flowing water at l is shown in the form:

$$\tau = al^{3/8} \sin^{13/16} \theta. \quad (4.8)$$

Equation (4.8) has the same form with equation (2.7). From formulas for tractional load, the equation of continuity of sediment discharge and equation (2.13), an equation of erosion having the same form with equation (2.12) can be derived. In this case the values of m and n are supposed to be smaller. The value of K is large as q' is much greater than q . Erosional mechanism of mass movement flowing down in the channel is also shown by equation (2.19). The volume of debris flow increases gradually as it flows down in the channel, gaining part of bed material and debris which is supplied from valley walls. Since the debris flow runs down in the limited channel, it is easily recognized that the erosive force acting on a unit width of the bed is a function of l .

4.2 Change in Longitudinal Profile of Radial Valley

The average slope profile previously considered is an imaginary profile obtained by burying valleys with the materials of ridges and by smoothing the slopes uniformly. Then, we measure the amount of change of longitudinal profile of a radial valley which is an actual landform, and consider the applicability of equation B to it. Radial

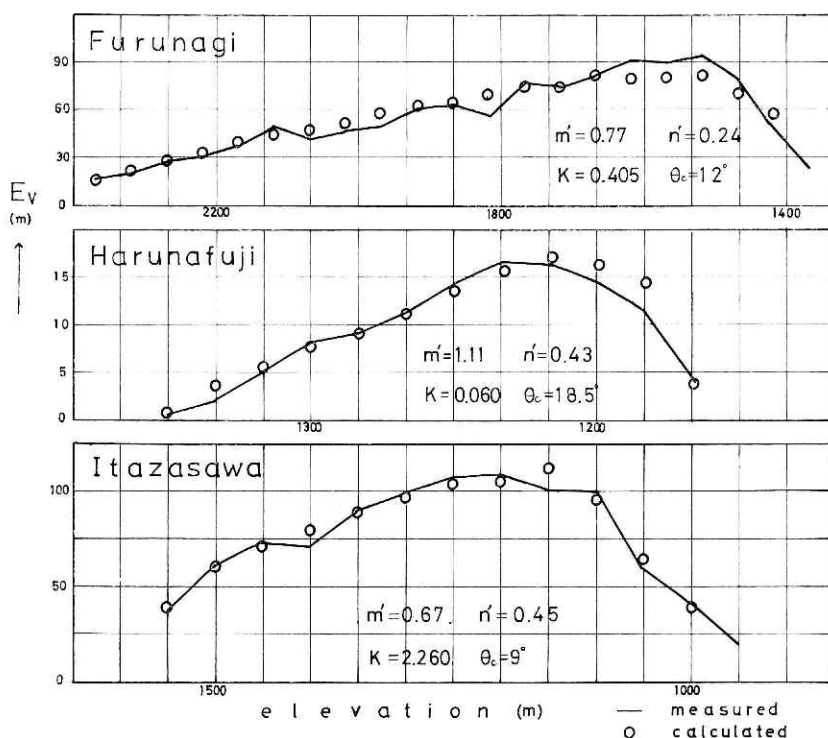


Fig. 15. Comparison of measured amount of undercutting of valley bed with the calculated. Radial valley.

valleys here considered are Furunagi of Mt. Nantai, Itazasawa of Mt. Iwate and the three main valleys of the measured part of Mt. Harunafuji. The amount of undercutting of valley bed E_v was measured by the difference between initial slope profile and longitudinal profile of the valley bed.

The comparisons of the calculated values with the measured are shown in Fig. 15. In this case too, both the values agree with each other quite well. Correlation coefficients are 0.962 for Furunagi V., 0.977 for Itazasawa V. and 0.976 for Mt. Harunafuji. All of them are significant at the 0.001 level of probability. Thus, it can be clarified that the change of longitudinal profile of typical radial valley is carried on through the physical mechanism represented in equation B . The values of m' and n' are smaller in this case as supposed previously.

4.3 Development of Radial Valley

From the result that the changes of average slope profile and longitudinal profile of a radial valley are described by equation B , the following simple process of radial valley evolution can be assumed that the erosion process at a valley bed is dominant and the recession of valley walls proceeds in proportion to the amount of undercutting of the valley bed, keeping the similarity of the shape of the cross section. It can be recognized by observation of dissected volcanoes that the cross section of radial valley is generally V-shaped. Ishii (1970) observed that the development of the valleys at the south slope of Mt. Nantai had been carried on by parallel recession of valley walls keeping the V-shaped cross section. When the size of the valley is small, the erosion of valley walls is thought to be carried on mainly by spontaneous mass movement. In this case it is easily recognized that the parallel recession of valley walls takes place.

In the cases shown above, if the erosion of valley bed is carried on by the mechanism represented by equation B , the change of average slope profile also follows it in consequence, since the depth of the valley keeps its proportionality to the cross-sectional area. The applicability of equation B which is derived from assuming uniform sheet erosion throughout the slope to the highly dissected volcanoes, of which valleys are deep, can be explained by the assumption.

4.4 Change of Valley Bed of the Dashibara Valley

The periods of erosion of the slopes thus far considered are very long except for those of coal slag heaps, and the measurement of the amount of erosion by the morphometry was performed based on some assumptions. Then, we consider the erosion process of valley bed by using the data obtained by direct surveying. The surveying of the change of stream bed has been performed for many streams in Japan by the public organizations.

Judging from the character of the equation of erosion, the valley which satisfies the following conditions may be favorable for consideration: the gradient of the channel is relatively steep, artificial structures like debris barriers do not exist, the distance between stations is relatively short, the amount of the change of the bed is large. Namely, the valley where the effects of topographic factors are not offset seemingly by those of other factors is favorable. The Dashibara valley, a tributary of the Joganji river, is a rare case where the conditions are satisfied. The Dashibara valley has been formed by the redisection of mud flow terrace formed by the large-scale landslide of Mt. Tombi in 1858. Intense erosion is now in progress. The drainage area is about 4 km². In the lower course of the river, the terrace is deeply dissected and the valley wall shows a badland topography.

The surveyings of the deepest bed elevation were performed in 1951—1965.

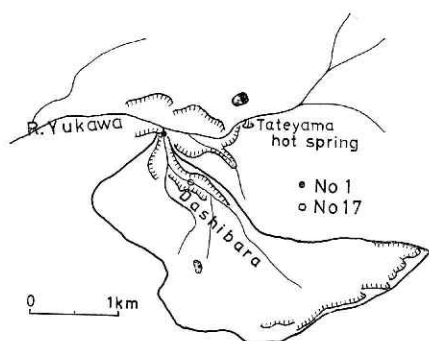


Fig. 16. Map of the Dashibara Valley.

station was calculated. By a profile obtained by averaging all of the profiles which were obtained by surveyings during the period, the gradient and the curvature of the bed were measured. In Fig. 17 longitudinal profiles in 1951, 1959 and 1964 are shown. The applicability of equation A can be confirmed by the existence of relative agreement of local variations of surveyed values with those of calculated ones. Therefore, the calculated value was obtained simply by determining the ratio of the coefficient of the term proportional to the gradient of slope I with that of curvature K_u by the method of least squares.

The relation between the surveyed values and the calculated in the period 1951—1959 during which surveying was performed every year is shown in Fig. 18. Correlation coefficient is 0.768, which is significant at the 0.001 level. It may be rather natural that the data are scattered fairly apart from a regression line, since the amount of change of the deepest bed elevation is taken as the amount of erosion depth, and the change of channel width and bed materials with locations and the effect of confluence with tributaries are neglected. Thus, it can be supposed that the change of

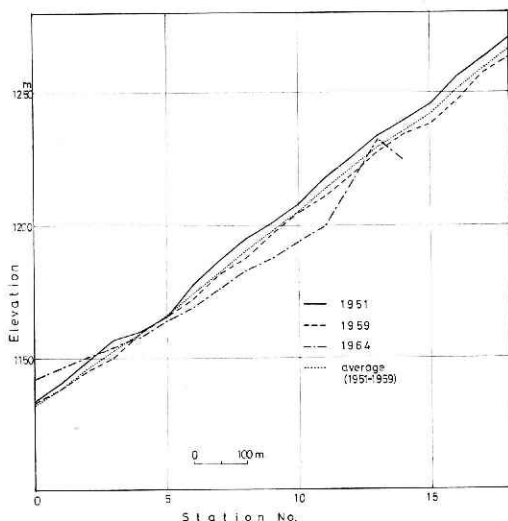


Fig. 17. Longitudinal profiles of the Dashibara Valley.

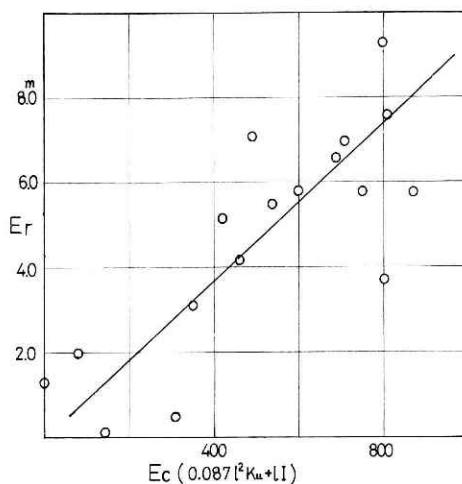


Fig. 18. Relation of surveyed amount of the change of valley bed (E_r) to the calculated (E_c), 1951—1959 (1).

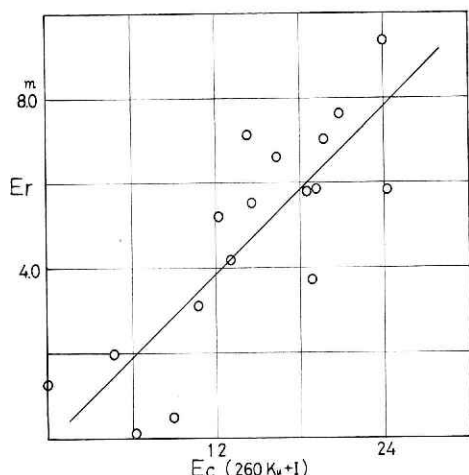


Fig. 19. Relation of surveyed amount of the change of valley bed (E_r) to the calculated (E_c), 1951–1959 (2).

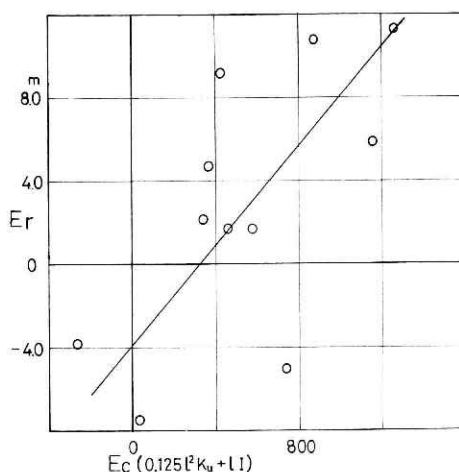


Fig. 20. Relation of surveyed amount of the change of valley bed (E_r) to the calculated (E_c), 1959–1964.

valley bed is carried on through the mechanism described by equation 4.

The relation of the surveyed values to the calculated ones obtained by putting l as constant is shown in Fig. 19. Highly significant correlation is obtained in this case too. The reason of the high correlation may be that at a short range of a channel located far in a large distance from the divide, the effect of l is seemingly eliminated, since the amount of lateral inflow is very little compared with that of the flow from upstream, and the increase of discharge by lateral inflow is partly offset by the increase of channel width. However, the factor of l must not be neglected, judging from the character of the phenomenon.

The result obtained by the data in 1959–1964, during which the valley bed rose by about 8 m near the mouth of the valley by debris flow, is shown in Fig. 20. Correlation coefficient is 0.622, which is significant at the 0.05 level. During the period the surveying was performed only two times, in 1959 and in 1964. Topographic factors were measured by the profile obtained by averaging these two profiles. The low correlation may be partly due to this. The data from the stations Nos. 13 to 17 were excluded because abnormal values are presented by the alteration of the standard elevation.

Debris flows have occurred frequently in the Dashibara valley. During the considered period, large-scale debris flows occurred in 1952 and 1963. Hence, the change of the valley bed is supposed to be carried on mainly by debris flow. Then, it may be assumed that the erosional mechanism of debris flow is described by equation 4, and that the fundamental process of erosion may be similar, independently of the period of erosion and the type of erosive agent.

5. ON THE ROLE OF INDEPENDENT TERMS OF GRADIENT AND CURVATURE OF SLOPES

5.1 Multiple Regression Analysis

As we have seen in the previous section, the equations of erosion in which the slope length is an important factor have been derived by assuming that dominant

erosive agents in humid regions are the tractive force of running water and the scouring force of materials of mass movement, and the calculated values obtained from the equations agree well with the measured values at many actual slopes. Therefore, the consideration presented thus far may be correct in general. However, there exist some erosion processes in which the transporting medium does not intervene and the location on the slope is not concerned.

Culling (1960) presents that the process of soil creep is related to the curvature of slope. Spontaneous mass wasting under the action of gravitational stress is affected by the gradient of slope. Several investigators such as Culling (1960), Scheidegger (1961), Hirano (1966) and so on studied the process of slope development by using mathematical models which were composed of independent terms of the gradient and curvature of slope. Since equations *A* and *B* apply well to actual processes, it may safely be concluded that these independent terms can be neglected. Then, in order to confirm this, we try to test the significance of the terms by multiple regression analysis.

A modified equation of erosion obtained by adding the terms of the gradient and curvature of slope into equation *B* is

$$E = \beta_1 l^{m'} (\sin \theta - \sin \theta_c)^{n'} + \beta_2 \sin \theta + \beta_3 \frac{d}{dx} (\tan \theta), \quad (5.1)$$

where unknown quantities are β_1 , β_2 , β_3 , m' and n' . Since the equation is non-linear with regard to unknown quantities, calculation by the method of least squares is very troublesome. However, the proportion of the first term of equation (5.1) is supposed to be so great that the values of m' and n' previously obtained can be given approximately. Thus, multiple regression coefficients β_1 , β_2 and β_3 are easily calculated.

Multiple regression analyses were carried on by using the data of the slopes previously considered where the accuracy of measurement was supposed to be high. Slopes taken up were the south slope of Mt. Nantai, Mt. Harunafuji, Mt. Maefurano and the Itazasawa valley of Mt. Iwate. They cover a fairly wide range of stages of erosion. The results of *t*-test of regression coefficients and simple correlation coefficients of the first terms r_1 are shown below. The number of the data is denoted by *N*. Data of the upper slope where the accuracy of measurement was supposed to be low were excluded. The inequality $t_n > t_{\alpha}(N-4)$ means that the regression coefficient of the *n*-th term is significant at the α level of significance. When the sign of the inequality is reverse, it means that the coefficient is not significant even at the α level. The values of m' and n' were set at 1.0 for simplification.

Nantai, south ($N=14$):

$$t_1 > t_{0.001}(10), t_2 < t_{0.1}(10), t_3 < t_{0.1}(10), \\ r_1 = 0.983;$$

Harunafuji ($N=10$):

$$t_1 > t_{0.001}(6), |t_2| > t_{0.05}(6), t_3 < t_{0.1}(6), \\ r_1 = 0.977;$$

Maefurano ($N=12$):

$$t_1 > t_{0.001}(8), t_2 < t_{0.1}(8), t_3 < t_{0.1}(8), \\ r_1 = 0.986;$$

Itazasawa ($N=11$):

$$t_1 > t_{0.001}(7), t_2 < t_{0.1}(7), t_3 < t_{0.1}(7), \\ r_1 = 0.974.$$

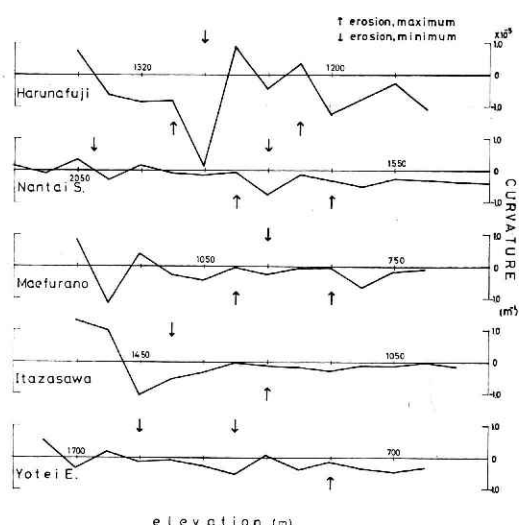


Fig. 21. Relation of curvature of slope to the change of the amount of erosion depth.

and curvature of slope by themselves are very little, at least not important factors of erosion. Their function may be limited to local processes of transportation and deposition. A dominant process may be represented by a function in which the factor of location on the slope is an important one. When we study actual processes of slope erosion, it is necessary to consider not only the kind of process but also the magnitude and function of it on actual slopes. As shown earlier, abrasion due to materials of mass movement is considered as a dominant process. At the location of its occurrence, spontaneous mass movement must take place. Therefore, the term of gradient of slope cannot be neglected.

5.2 Relation of Curvature of Slope to the Change of the Amount of Erosion

Curvature of slope by itself could not be recognized as a significant factor of erosion. It may be related to erosion in the form as shown in equation *A*. There are some kinds of processes in which the curvature of slope by itself is an important factor like soil creep. Then, we consider the relation of the curvature of slope to the change of the amount of erosion. In Fig. 21 the altitudinal changes of the curvature of slope are shown. The local minimum point of erosion depth is shown with a downward arrow and the local maximum with an upward arrow. From the figure the following tendency can be noticed, that is, at the place of local maximum or minimum of erosion, the curvature of slope is also at its local maximum or minimum, though the reverse is not always true. Namely, it represents that at convex or less concave parts of the slope, erosive action is relatively strong compared with the neighboring, more concave parts, and hence the amount of erosion is relatively large. At concave parts the reverse is true. By this process the ruggedness of slope tends to be eliminated.

6. PROCESSES OF SLOPE DEVELOPMENT

6.1 Derivation of the Equation of Slope Development

Equations *A* and *B* of erosion give the amount of change of a slope in a given time. From these equations, equations which represent the change of the amount of slope

In every case, regression coefficients of the terms of gradient and curvature of slope are not significant even at the 0.1 level, that is, null hypothesis can not be rejected. The coefficient of the second term of Mt. Harunafuji is significant at the 0.05 level, but significance cannot be admitted, since the value is minus, which is physically meaningless. On the other hand, though simplified as $m'=n'=1$, all of the regression coefficients of the first terms are significant at the 0.001 level. Simple correlation coefficients are more than 0.97.

Although the number of data is small and the accuracy of it may not be very high, from the result obtained it can be supposed that the gradient

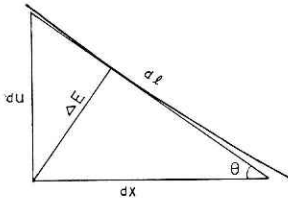


Fig. 22. Method of calculation.

degradation with time can be derived. The rate of erosion $\Delta E/\Delta t$ is given by equation (2.12) as follows:

$$\frac{\Delta E}{\Delta t} = -k_1 l^m \frac{d}{dl} (-\sin \theta)^n - k_2 l^{m-1} (-\sin \theta)^n. \quad (6.1)$$

The sign of the gradient of the slope is minus, since the slope profile is represented by the curve sloping down toward x . Since ΔE is measured normally to the slope, we convert it into the amount of degradation. By Fig. 22,

$$\begin{aligned} \Delta E &= du \cos \theta \\ &= du \left\{ 1 + \left(\frac{du}{dx} \right)^2 \right\}^{-1/2} \end{aligned} \quad (6.2)$$

is obtained, where u is vertical distance, and x horizontal distance. Using the relation

$$\sin \theta = \frac{du}{dx} \left\{ 1 + \left(\frac{du}{dx} \right)^2 \right\}^{-1/2}, \quad (6.3)$$

and representing with a partial differential equation, as u is a function of x and t , equation (6.1) is transformed into

$$\begin{aligned} \frac{\partial u}{\partial t} &= -k_1 l^m \frac{\partial}{\partial l} \left[-\frac{\partial u}{\partial x} \left\{ 1 + \left(\frac{\partial u}{\partial x} \right)^2 \right\}^{-1/2} \right]^n \left\{ 1 + \left(\frac{\partial u}{\partial x} \right)^2 \right\}^{1/2} \\ &\quad - k_2 l^{m-1} \left[-\frac{\partial u}{\partial x} \left\{ 1 + \left(\frac{\partial u}{\partial x} \right)^2 \right\}^{-1/2} \right]^n \left\{ 1 + \left(\frac{\partial u}{\partial x} \right)^2 \right\}^{1/2}, \end{aligned} \quad (6.4)$$

where

$$dl = dx \left\{ 1 + \left(\frac{du}{dx} \right)^2 \right\}^{1/2}, \quad l = \int_0^x \left\{ 1 + \left(\frac{du}{dx} \right)^2 \right\}^{1/2} dx.$$

Equation (6.4) is a complicated non-linear partial differential equation, which is inconvenient for theoretical consideration. And, for simplification we assume as follows: supposing that the flow caused by rainfall is the dominant agent, l can be replaced with x because the amount of rainfall is proportional not to l but to x , and as $(\partial u/\partial x)^2$ is relatively small, in general, on mountain slopes, we put $1 + (\partial u/\partial x)^2 \approx 1$ and by using the result of measurement, we put $n=1$ in order to make linear. Thus, equation (6.4) can be simplified as follows:

$$\frac{\partial u}{\partial t} = k_1 x^m \frac{\partial^2 u}{\partial x^2} + k_2 x^{m-1} \frac{\partial u}{\partial x}. \quad (6.5)$$

Here, this is called equation A of slope development. It may be allowed to put $m=2$ when we make theoretical consideration. Equation (6.5) is an extended equation of diffusion. The result is reasonable since the slope erosion is a kind of non-reversible process of diffusion of the material forming the slope.

Culling (1960) presented the following equation of slope development by a consideration of the process of soil creep:

$$\frac{\partial u}{\partial t} = \kappa \frac{\partial^2 u}{\partial x^2}.$$

Scheidegger (1961) derived

$$\frac{\partial u}{\partial t} = -\frac{\partial u}{\partial x} \sqrt{1 + \left(\frac{\partial u}{\partial x} \right)^2}$$

by a simple deductive consideration of physical process. Hirano (1966) proposed the following equation as a mathematical model of slope development of a finite mountain:

$$\frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial x^2} + b \frac{\partial u}{\partial x}.$$

Most of the models proposed so far are composed of independent terms of the gradient and curvature of slope, and none of these have a factor of position. Since the magnitude of erosive agents is a function of location, mathematical models of slope development should have the term incorporated. Takeshita (1963) obtained the following equation by the observation of landslide experiments:

$$\frac{\Delta E}{\Delta t} = (b + a\sqrt{h}) \sin \theta,$$

where h is vertical distance from the upper surface of the slope. This has a factor of location.

In the same way as in the case of equation A , equation B is transformed into

$$\frac{\Delta E}{\Delta t} = -Kl^{m'}(-\sin \theta - I_c)^{n'}. \quad (6.6)$$

Then, from equation (6.6) the following equation is derived:

$$\frac{\partial u}{\partial t} = Kx^{m'} \left(\frac{\partial u}{\partial x} + I_c \right). \quad (6.7)$$

This is called equation B of slope development.

By equations A and B , slope deformation does not occur at the top of the slope where $x=0$. In a previous consideration, abrasion due to materials of mass movement was taken up as a dominant transporting process. At the point of occurrence of mass movement, spontaneous landslide must take place. Then, by adding the term of the gradient of the slope to equation (6.7),

$$\frac{\partial u}{\partial t} = K_1 x^{m'} \left(\frac{\partial u}{\partial x} + I_c \right) + K_2 \frac{\partial u}{\partial x} \quad (6.8)$$

is obtained as an extended equation of equation B . The extension of the equation is made only about equation B which is easily solved analytically. Since the significance of the term of the gradient of slope could not be admitted, it may represent the process much better to make only the top of the slope degrade by giving a proper boundary condition supposing that the top of the slope is a singular point. Erosive factors like geology, climate, vegetation and so on may be represented in a function of location, and as shown previously, θ_c is a function of erosional stage, hence that of time. Then, equation (6.8) can be generalized as follows:

$$\frac{\partial u}{\partial t} = f_1(x, u)x^{m'} \left\{ \frac{\partial u}{\partial x} + g(t) \right\} + f_2(x, u) \frac{\partial u}{\partial x}. \quad (6.9)$$

6.2 Application of the Equations of Slope Development to Actual Processes

As equations A and B of slope development were derived by considerable simplification, it is necessary to confirm their validity by applying them to actual processes of slope deformation. We choose the south slope of Mt. Nantai and Mt. Harunafuji as examples. The amount of slope degradation D was measured simply by $D = R \tan \theta$. Calculated values of D was obtained by numerical calculation of the difference equations which were derived from equations A and B . The gradient and the curvature of the slope at x_m and at $t=n\Delta t$ are approximated by

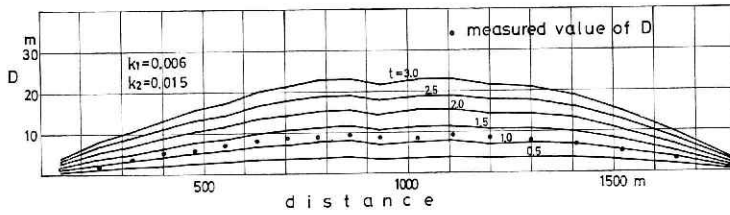


Fig. 23. Comparison of measured amount of degradation of Mt. Nantai with the calculated by equation A of slope development.

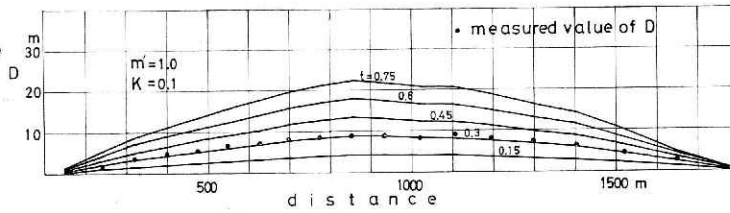


Fig. 24. Comparison of measured amount of degradation of Mt. Nantai with the calculated by equation B of slope development.

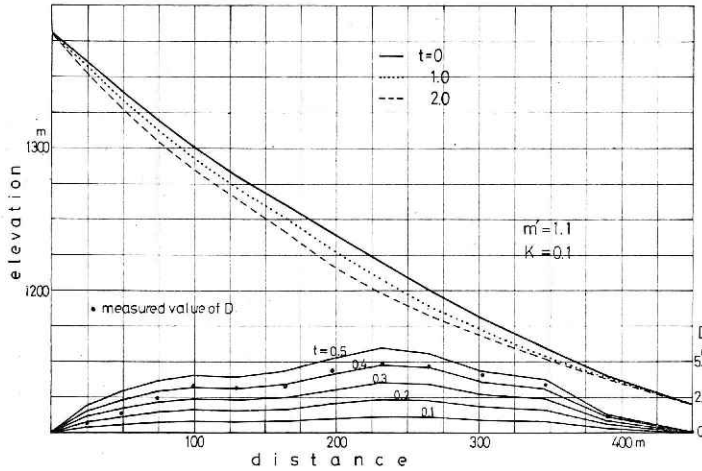


Fig. 25. Calculated average profiles of Mt. Harunafuji and comparison of measured amount of degradation with the calculated by equation B of slope development.

$$\left(\frac{\partial u}{\partial x}\right)_{m,n} = \frac{u_{m+1,n} - u_{m-1,n}}{x_{m+1,n} - x_{m-1,n}}, \quad (6.10)$$

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{m,n} = \frac{1}{2(x_{m+1,n} - x_{m-1,n})} \left(\frac{u_{m+1,n} - u_{m,n}}{x_{m+1,n} - x_{m,n}} - \frac{u_{m,n} - u_{m-1,n}}{x_{m,n} - x_{m-1,n}} \right). \quad (6.11)$$

Similarly the rate of slope degradation during Δt is

$$\left(\frac{\partial u}{\partial t}\right)_{m,n} = \frac{u_{m,n+1} - u_{m,n}}{\Delta t}. \quad (6.12)$$

By the difference equations obtained by substituting these relations into equations A and B, the slope profile at the time step $n+1$ can be calculated by that at the time step n if the initial profile is given.

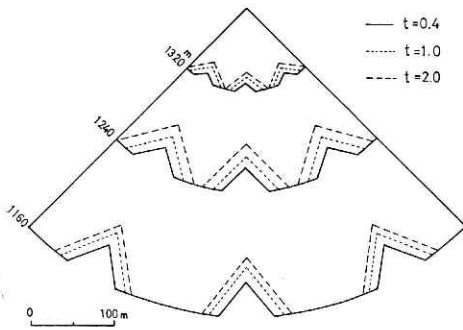


Fig. 26. Diagrammatic map of Mt. Harunafuji in future.

A of slope development may be confirmed.

In Figs. 24 and 25, the case of equation B is shown. The value of m' of Mt. Nantai is set at 1.0 and that of Mt. Harunafuji at 1.1 as a result of measurement. Calculation was performed under the condition that the degradation of the top and bottom of the slope is zero. In the case of Mt. Nantai, the measured value agrees quite well with the calculated at $t=0.3$. In the case of Mt. Harunafuji, the measured value agrees with the calculated at $t=0.4$. By the results the validity of equation B may be confirmed.

Calculated average profiles of Mt. Harunafuji at $t=1.0$ and $t=2.0$ are also shown in Fig. 25. A diagrammatic topographical map of Mt. Harunafuji in future can be drawn by the profiles. Figure 26 shows the map obtained by the supposition that valleys lie at regular intervals, their cross section being V-shaped and valley-side angle being 30° . At the altitude of 1,320 m the initial surface begins to disappear at $t=2.0$. At the lower part of the slope the initial surface is preserved over a long period of time. Assuming that Mt. Harunafuji is as old as Mt. Futatsudake, a neighboring lava dome which is supposed to have been formed 1,400 years ago, the profile at $t=1.0$ shows that 2,000 years later and the profile at $t=2.0$, 6,000 years later on the condition that climatic factors remain constant.

6.3 Processes of Slope Development by Equation A

Putting $m=2$, equation (6.5) is rewritten as follows:

$$\frac{\partial^2 u}{\partial x^2} + \frac{k_2}{k_1 x} \frac{\partial u}{\partial x} - \frac{1}{k_1 x^2} \frac{\partial u}{\partial t} = 0. \quad (6.13)$$

By the transformation $x = \exp(\sqrt{k_1} X)$, equation (6.13) is reduced to

$$\frac{\partial^2 u}{\partial X^2} - \frac{\partial u}{\partial t} + \frac{k_2 - k_1}{\sqrt{k_1}} \frac{\partial u}{\partial X} = 0. \quad (6.14)$$

By the transformation

$$u = U \exp(KX), \quad K = \frac{k_1 - k_2}{2\sqrt{k_1}},$$

equation (6.14) is reduced to

$$\frac{\partial^2 U}{\partial X^2} - \frac{\partial U}{\partial t} - K^2 U = 0, \quad (6.15)$$

where the term of $\partial U / \partial X$ is eliminated, but the term of U is appearing. Then, further transformation $U = V \exp(-K^2 t)$ is carried on for elimination of the term of U . Thus,

In Fig. 23 solid lines show the change of calculated values of D with time which are obtained from equation A . Calculation was performed by taking $m=2$, $\Delta t=0.0005$, $k_1=0.006$ and $k_2=0.015$ using an electronic computer. The values of k_1 and k_2 are those obtained previously. Since the shore of Lake Chuzenji lies at $x=2,260$, the boundary condition $(\partial u / \partial t)_{x=2260}=0$ is given. Black circles show the measured amount of D . The measured amounts of D change in parallel with the calculated. The measured value is in the stage of $t=1.2$. Thus, the validity of equation

we obtain

$$\frac{\partial^2 V}{\partial X^2} - \frac{\partial V}{\partial t} = 0. \quad (6.16)$$

This is the diffusivity equation of one dimension, whose solution is well known. The solution which satisfies the initial condition $V_{t=0}=g(X)$ is given by

$$V = \frac{1}{2\sqrt{\pi t}} \int_{-\infty}^{+\infty} g(\xi) \exp \left\{ -\frac{(\xi - X)^2}{4t} \right\} d\xi. \quad (6.17)$$

Then, we give $u_{t=0}=f(x)$ for the initial condition expressed by u and x . This initial condition is rewritten as

$$V = \exp(-KX) f\{\exp(\sqrt{k_1}X)\}. \quad (6.18)$$

Then, by the inverse transformation

$$\begin{aligned} X &= \frac{1}{\sqrt{k_1}} \ln x, \\ V &= \exp(K^2 t) x^{-K/\sqrt{k_1}} u, \end{aligned} \quad (6.19)$$

the solution of equation (6.5) with initial condition $u_{t=0}=f(x)$ is given by

$$\begin{aligned} u &= \exp(-K^2 t) x^{K/\sqrt{k_1}} \frac{1}{2\sqrt{\pi t}} \int_{-\infty}^{+\infty} \exp(-K\xi) f\{\exp(\sqrt{k_1}\xi)\} \\ &\quad \times \exp \left\{ -\frac{(\xi - \ln x^{1/\sqrt{k_1}})^2}{4t} \right\} d\xi, \end{aligned} \quad (6.20)$$

where $x \geq 0$, since it is the distance from the divide.

Let us consider the general processes of slope deformation with the lapse of time which are obtained by solving equation A under various conditions. In case A.1 of slope development, the initial condition is given by the following exponential curve:

$$u = 10 \exp(-0.1x - 0.4), \quad (6.21)$$

which has a similar form with the profiles of general strato-volcanoes. The change of slope profile with time is obtained by numerical integration of equation (6.20) as shown in Fig. 27. The rate of slope degradation decreases with the lapse of time. In the early stage of erosion, deposition takes place at the foot of the slope with decreasing speed. At about $t=0.4$, deposition ceases and thereafter gradual redissection is carried on. As a result of measurement, $R_{\max}/r=0.05-0.1$ is obtained as a rough criterion which represents the stage when the initial surface begins to disappear at a certain altitude. In Fig. 27 the stage corresponds to the profile at $t=0.2-0.4$. Although the applicability of the equations of erosion is not confirmed by morphometric measurement in the later stages, profiles at larger values of t are presented to show the change of slope profile clearly in the figure. Therefore, we do not necessarily assert that the upper slope becomes so steep as shown in the figure. Around the summit the slope degradation may take place by spontaneous mass movement accelerated by steepening of the slope. Nevertheless, the upper slope may become steeper after all, since the erosive force is thought to be relatively weak there. This may be recognized by observation of highly dissected volcanoes. The relative ratio of the coefficient of the second term to that of the first, k_2/k_1 , is set at 1.5, in order to make agreement with observed facts. In case $k_2/k_1=2.0$ which is obtained directly from equation (2.12), a valley is prolonged to a place where the slope is too gentle. Calculated values of the ratio are 3.3 at Mt. Harunafuji and 2.4 at Mt. Nantai. The difference may be due to local variations of erosive factors, especially that of infiltration capacity at actual mountain slopes.

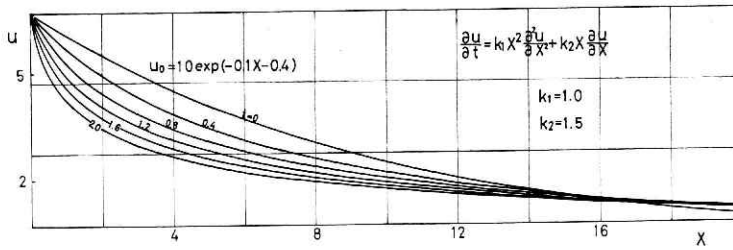


Fig. 27. Slope development in case A.1. Initial profile has a similar form with the profiles of general strato-volcanoes.

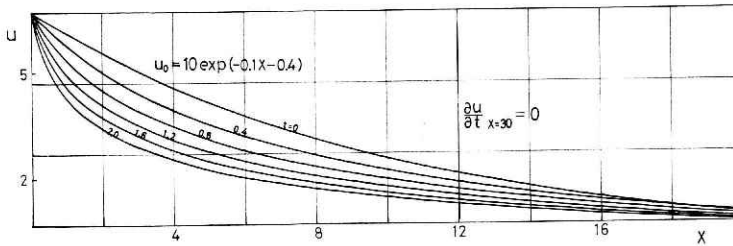


Fig. 28. Slope development in case A.2. Development of slope with the boundary condition $(\partial u / \partial t)_{x=30} = 0$.

As is often the case, there is a river at the foot of a mountain and the material from the slope is carried away by the river. This situation is expressed by the boundary condition that the slope deformation does not occur at the bank of the river. The solution of equation (6.16) with the boundary conditions $V_{t=0} = g(X)$, $V_{x=L} = 0$ is

$$V = \frac{1}{2\sqrt{\pi t}} \int_0^\infty g(L - \xi) \left[\exp \left\{ -\frac{(L - \xi - X)^2}{4t} \right\} - \exp \left\{ -\frac{(L + \xi - X)^2}{4t} \right\} \right] d\xi. \quad (6.22)$$

Then, the solution of equation (6.5) which satisfies the boundary conditions $u_{t=0} = f(x)$ and $u_{x=L} = 0$ is obtained by the same inverse transformation given by equation (6.19) as follows:

$$\begin{aligned} u = & \exp(-K^2 t) x^{K/\sqrt{k_1}} \frac{1}{2\sqrt{\pi t}} \int_0^\infty \exp(-\ln l^{K/\sqrt{k_1}} + K\xi) \\ & \times f\{\exp(\ln l - \sqrt{k_1}\xi)\} \left[\exp \left\{ -\frac{(\ln l^{1/\sqrt{k_1}} - \ln x^{1/\sqrt{k_1}} - \xi)^2}{4t} \right\} \right. \\ & \left. - \exp \left\{ -\frac{(\ln l^{1/\sqrt{k_1}} - \ln x^{1/\sqrt{k_1}} + \xi)^2}{4t} \right\} \right] d\xi. \end{aligned} \quad (6.23)$$

Figure 28 shows case A.2 where the boundary condition $(\partial u / \partial t)_{x=30} = 0$ is given. Slope deformation at the foot of the slope is small within the limit of small values of t in which the disappearance of the initial slope does not occur. This case is more actual than case A.1 where the amount of deposition at the foot is large. At an actual foot of a mountain the redissection of a depositional surface does not occur so intensely as shown in Fig. 28 due to high permeability. Zone of erosion is extended gradually to the lower part of the slope. This extension is confirmed by the measurement.

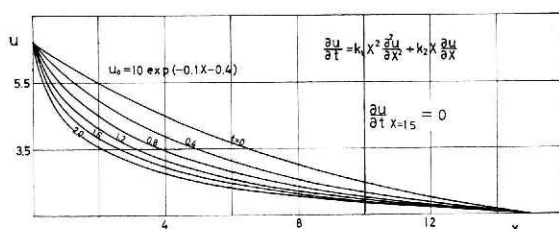


Fig. 29. Slope development in case A.3. Development of slope with the boundary condition $(\partial u / \partial t)_{x=15} = 0$.

Case A.3 shows the change of a slope profile under the boundary condition $(\partial u / \partial t)_{x=15} = 0$. The result is shown in Fig. 29. The gradient of the initial profile at $x=15$ is 8.5 degrees which is nearly equal to the value of θ_c of volcanoes in their early mature stage. Therefore, the result can be compared with that by equation B.

Here, we consider the change of the amount of slope deformation with time by using the profiles shown in Fig. 29. Supposing that the shape of the mountain is conical like uniform strato-volcanoes, the degree of the decrease of the volume was calculated for each profile. In earlier stages the rate of dissection is large, but after about one-third of the mountain is eroded the rate decreases rapidly. Suzuki (1969) stated that the erosion rate of volcanoes changes with time in the way represented by a logarithmic curve. Nevertheless, the curve shown in Fig. 30 is not logarithmic. The degrees of erosion of the volcanoes whose ages are estimated were measured. The absolute value of t in Fig. 30 can be obtained by plotting the values on the curve. In this case $t=1.0$ corresponds to about 2.3×10^5 years. Then, after 3×10^5 years half of the volume of the mountain is eroded and after a million years about 80% of the volume is eroded away. These values may be reasonable considering the fact that the volcanoes formed in Tertiary Period are not existing. Average rate of erosion of young volcanoes is calculated to be about 1.0 mm/year.

Case A.4 shows the change of a slope profile which is convex at its upper part and strongly concave at its lower part. Initial and boundary conditions are given by

$$u_0 = 5 \exp \{-(0.15x)^2\}, \quad \left(\frac{\partial u}{\partial t} \right)_{x=15} = 0. \quad (6.24)$$

The result is shown in Fig. 31. Slope degradation proceeds rapidly at the upper convex part, because the curvature of the slope is positive there, and the first term of equation A gives the amount of degradation. Therefore, the convex part soon disappears and a steep concave slope is formed. At the lower concave part, deposition occurs in an earlier stage. As a result, concavity becomes weak and redissection begins.

Case A.5 shows the change of a straight slope. Some of volcanic cones have nearly straight slope. Initial and boundary conditions are given by

$$u_{t=0} = 5 - 0.5x, \quad 10 \geq x \geq 0; \quad u_{t=0} = 0, \quad x > 10; \quad u_{x=15} = 0. \quad (6.25)$$

The result is shown in Fig. 32. In the initial stage the aggradation of the slope takes place at $x=10$ which is a flexing point. Then, the effect extends upwards and downwards, and the zone of deposition is enlarged. At a maximum, the upper limit of depositional zone is situated at $x=7.5$. Thereafter, redissection is carried on and the degradation of the slope takes place in the entire range of the slope. This process

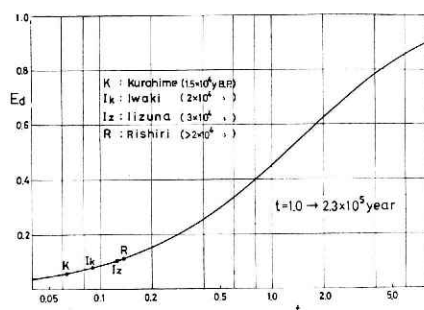


Fig. 30. Change of the degree of erosion (E_d) with time.

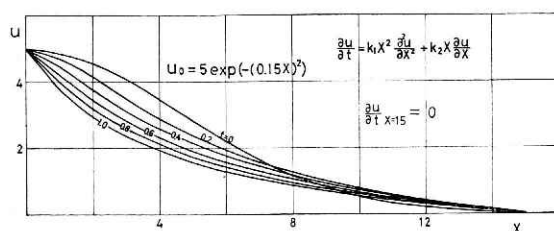


Fig. 31. Slope development in case A.4. Initial slope is convexo-concave.

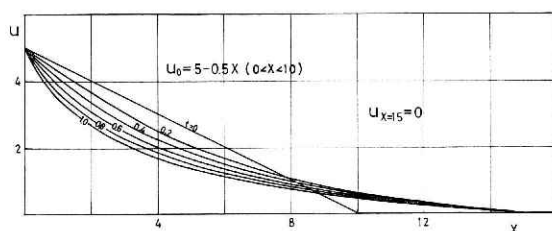


Fig. 32. Slope development in case A.5. Initial slope is straight.

cannot be represented in the figure, because the amount of change is very small in the stages of small values of t .

The cases shown above suggest that in highly dissected stages a similarly concave slope is formed irrespective of the shape of initial profile. As shown in Fig. 33 the profiles of large values of t are almost linear in semilog papers. Therefore, graded profile of average slope formed by erosion and that of longitudinal valley profile can be approximated by a logarithmic curve. This is confirmed by the measurements of radial valleys.

By equation A the degradation of the top of the slope cannot be shown, and an unnatural steep slope is formed in the stages of large values of t . The summit can be denuded by giving the boundary condition $u_{x=0} = u_0 - f(t)$, but the physical meaning is not clear and it is difficult to give a realistic rate of degradation. At steep parts of slopes, degradation due to spontaneous mass movement may be a dominant process. It can be supposed that when the slope becomes too steep, spontaneous mass movement will occur frequently and the angle of the average slope will be reduced to a certain reposed angle in the long run. By morphometric measurements, 39° is obtained for the slope angle of the upper part of Ohsawa of Mt. Fuji and 37.5° for that of Mt. Rishiri and that of Furunagi of Mt. Nantai. Then, 40° may be a realistic value for the reposed angle.

Case A.6 shows the process of slope deformation obtained from equation (6.5) under the condition that the average angle of the slope between $x=0$ and $x=1$ is reduced to 40° by the degradation of the top of the slope when it exceeds 40° . Equation (6.5) is approximated by

$$\begin{aligned} \frac{u_{m,n+1} - u_{m,n}}{\Delta t} = & k_1 \{(m-1)\Delta x\}^2 \frac{u_{m+1,n} - 2u_{m,n} + u_{m-1,n}}{(\Delta x)^2} \\ & + k_2 (m-1)\Delta x \frac{u_{m+1,n} - u_{m-1,n}}{2\Delta x}. \end{aligned} \quad (6.26)$$

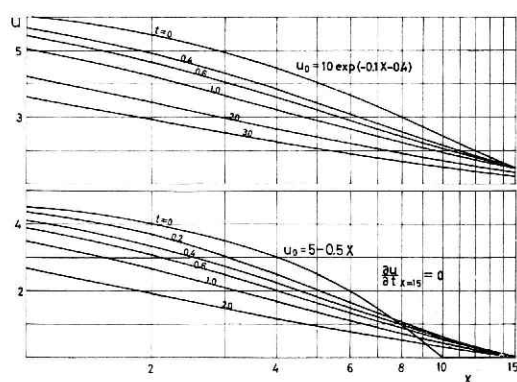


Fig. 33. Graded profiles of erosional landforms.

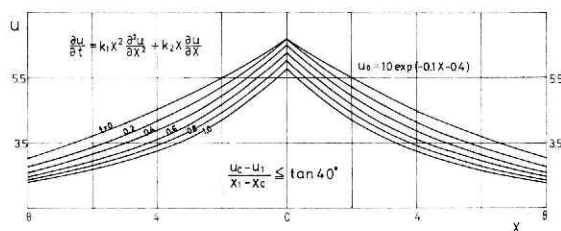


Fig. 34. Slope development in case A.6. Development of slope in case the summit is degraded.

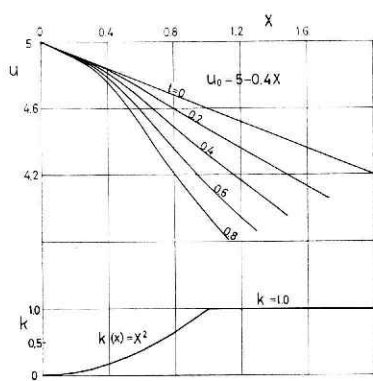


Fig. 35. Slope development in case A.7. Formation of convex hilltop.

By equation (6.26) the value of u at the time step $(n+1) \Delta t$ can be obtained from that of u at the time step $n \Delta t$. Calculation was performed by putting $k_1=1$, $k_2=1.5$, $\Delta x=0.5$ and $\Delta t=0.0005$. The result of numerical calculation is shown in Fig. 34.

Taking a microscopic view, a convex slope is generally formed at the top of the slope. This may be caused by the fact that the condition $\tau \gg \tau_c$, which is a premise of derivation of the equation of erosion, is not satisfied near the divide. The width of the belt of no erosion differs with the intensity of rainfall. At the time of heavy rain, erosion occurs from just below the divide. Frequency distribution of rainfall intensity is generally shown by an exponential curve. Therefore, near the divide, the cumulative amount of effective runoff in the long period of time increases rapidly with increasing distance from the divide. This indicates that the value of m of equation (6.5) is much larger within a certain limit of small values of x . Figure 35 shows case A.7 in which m is set at 4 within the range of $0 < x < 1$. This is identical with replacing k with kx^2 . Calculation was performed by equation (6.26) with the initial condition $u_0=5-0.4x$. Convex slope is clearly formed near the top of the slope.

In the cases shown above, the divide does not move since the slopes like those of volcanic cone

are supposed. Then, we consider the cases where the divide recedes. We suppose that an available catchment area of the valley, which is formed at the side slope of a plateau, is limited within a certain distance from the head of the valley, and the area is enlarged with headward erosion of the valley in proportion to the amount of the extension. Equation of slope development in this case can be given by modifying equation A as follows:

$$\frac{\partial u}{\partial t} = k_1(x-l+at)^2 \frac{\partial^2 u}{\partial x^2} + k_2(x-l+at) \frac{\partial u}{\partial x}, \quad (6.27)$$

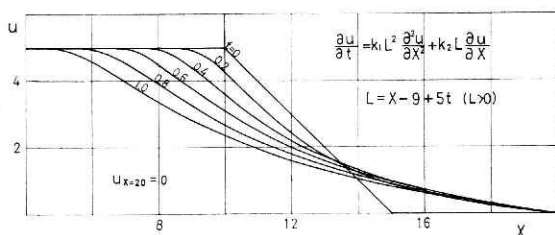


Fig. 36. Slope development in case A.8. Development of slope in case the divide recedes.

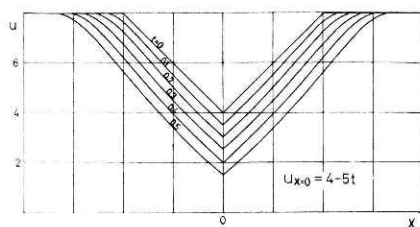


Fig. 37. Slope development in case A.9. Evolution of V-shaped valley.

where l is a constant which represents the length of the flat land surface and is much greater than at . The rate of headward extension is supposed to be proportional to t . The value of $x-l+at$ is positive since it represents the distance from the divide.

In case A.8, the process of slope deformation obtained from equation (6.27) under the initial condition

$$u_0 = 5, 0 < x < 10; u_0 = 5 - x, 10 \leq x \leq 15; u_0 = 0, 15 < x; \quad (6.28)$$

and the boundary condition $u_{x=20}=0$ is considered. Equation (6.27) is approximated by the following difference equation:

$$\begin{aligned} \frac{u_{m,n+1} - u_{m,n}}{\Delta t} = & k_1 \{ (m-1)\Delta x - l + a(m-1)\Delta t \}^2 \frac{u_{m+1,n} - 2u_{m,n} + u_{m-1,n}}{(\Delta x)^2} \\ & + k_2 \{ (m-1)\Delta x - l + a(m-1)\Delta t \} \frac{u_{m+1,n} - u_{m-1,n}}{2\Delta x}. \end{aligned} \quad (6.29)$$

Calculation was performed by putting $a=5$, $l=9$, $k_1=1$, $k_2=1.5$, $\Delta x=0.5$ and $\Delta t=0.0005$. The result of numerical calculation is shown in Fig. 36. This may be taken as a model of the erosion of a plateau. The top of the slope becomes rounded.

Case A.9 is a model which shows the development of a radial valley obtained from equation (6.27). The initial surface between radial valleys is flat and the process of valley wall recession is similar to that of case A.8. A supposition on the process of valley evolution has been shown in the previous chapter. Here, following the supposition, we consider the case where the valley bed degrades at a constant rate, and in proportion to it the top of the valley wall recedes. Figure 37 shows the change of valley-side slope obtained by numerical calculation of equation (6.29) under the same condition as in case A.8. Boundary condition is given by $u_{x=0} = 4 - 5t$. The valley wall develops almost always keeping the similarity of the shape as supposed previously.

6.4 Processes of Slope Development by Equation B

As equation B of erosion does not satisfy the condition of continuity of slope material, the curves obtained from the equation represent an unnatural deposition at the lower slope where the gradient is smaller than I_c . However, since the calculated values obtained from equation B agree quite well with the measured ones at many moderately dissected slopes, equation B of slope development may be an available equation which represents actual process within the limit of small values of t . Equation B is convenient for theoretical consideration as it can be solved easily under complicated conditions.

Equation (6.9) can be solved by using the characteristic curve. A supplementary equation is given by

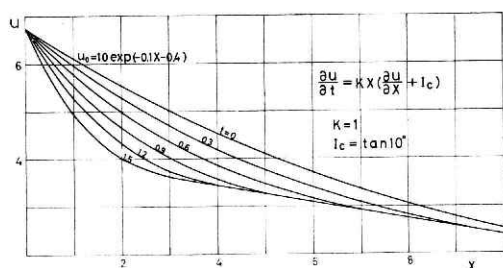


Fig. 38. Slope development in case B.1.
Initial profile has a similar form with the profiles of general strato-volcanoes.

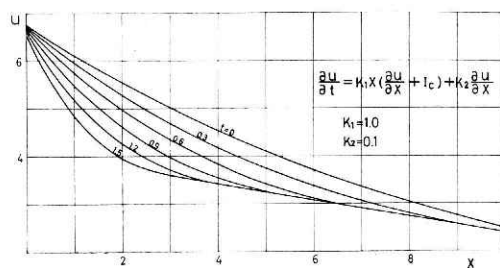


Fig. 39. Slope development in case B.2.
Development of slope in case the summit is degraded.

$$\frac{dt}{1} = -\frac{dx}{f_1(x, u)x^{m'} + f_2(x, u)} = \frac{du}{f_1(x, u)g(t)x^{m'}}. \quad (6.30)$$

The solution can be obtained by solving these two differential equations under an arbitrary initial condition.

Case B.1 is the simplest case where equation (6.7) is used and m' is set at 1. The solution of equation (6.7) with the initial condition $t=0$, $x=x_0$, $u=u_0$ is given by

$$\begin{aligned} u &= -I_c(x - x_0) + u_0, \\ x_0 &= xe^{Kt}. \end{aligned} \quad (6.31)$$

In Fig. 38 the change of slope profile with the lapse of time obtained by calculating equation (6.31) with $K=1$ and $I_c=0.176$, and giving equation (6.21) for the initial profile. As the initial slope has a similar form with a typical strato-volcano and the values of m' and n' are nearly 1 at most of the measured volcanoes, case B.1 may give a good approximation of the processes of slope development of volcanoes within the limit of small values of t . Profiles at larger values of t are shown in figures to show clearly the change of profile. In the case of equation B which cannot represent well the depositional phenomena at the foot of the slope, curves are drawn on the supposition that the aggradation of the slope does not occur in the considered range of the slope in the long run, namely, deposited materials are carried away from it after all. The curves in Fig. 38 bear the resemblance to those in Fig. 29. The maximum point of degradation migrates toward the top of the slope, keeping the altitude almost constant. Hence, the profile becomes more concave with time. Longitudinal profile of the valley in the slope may be much more concave and at the points downstream a nearly steady state may be realized.

In case B.2, the process described by equation (6.8) is presented. Putting $m'=1$ and giving equation (6.21) as the initial condition, the solution of equation (6.8) is given by

$$\begin{aligned} u &= -I_c(x - x_0) - \frac{K_2 I_c}{K_1} \ln \frac{K_1 x + K_2}{K_1 x_0 + K_2} + u_0, \\ x_0 &= \frac{K_1 x + K_2}{K_1} e^{K_1 t} - \frac{K_2}{K_1}. \end{aligned} \quad (6.32)$$

In Fig. 39 a series of curves obtained from equation (6.32) are shown. The ratio K_2/K_1 is determined based on the result obtained in the preceding chapter. Then, the effect of the size of the slope is eliminated. Degradation of the summit is very slow and the upper slope becomes steeper and more concave. In most of highly dis-

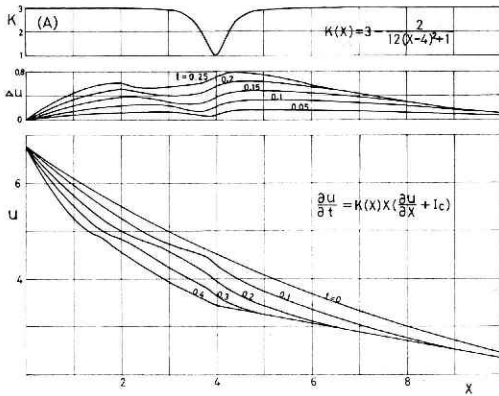


Fig. 40. Slope development in case B.3.
Hard rock exists at $x=4$.

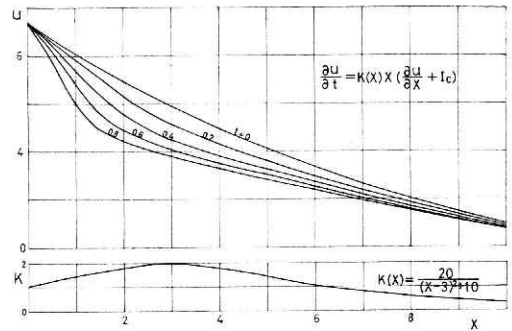


Fig. 41. Slope development in case B.3.
Weak rock exists around $x=3$.

sected volcanoes, pinnacles and cliffs exist around the summit. This may indicate that the degradation of the summit is relatively slow.

Cases B.3 and B.4 are examples in which the erodibility of the composing material of the slope differs from place to place. In case B.3, a hard rock like a volcanic neck exists vertically at $x=4$, and consequently erosion is delayed locally. This state is approximated by giving the curve shown in Fig. 40(A) for the value of $f_1(x, u)$ in equation (6.9). Taking $f_2(x, u)=0$, $g(t)=I_c$ and $m'=1$ for simplification, two independent solutions of equation (6.30) are

$$\begin{aligned} I_c x + u &= c_1, \\ t + \int \frac{1}{x f_1(x, u)} dx &= c_2, \end{aligned} \quad (6.33)$$

where c_1 and c_2 are arbitrary constants. Taking

$$f_1(x, u) = 3 - \frac{2}{12(x-4)^2 + 1}, \quad I_c = 0.176, \quad (6.34)$$

general solution of equation (6.9) is approximated by

$$\begin{aligned} \Phi \left[0.176x + u, t + \frac{1}{3} \ln x - \frac{1}{1728} \ln \left\{ (x-4)^2 + \frac{1}{36} \right\} \right. \\ \left. + \frac{1}{36} \arctan 6(x-4) \right] &= 0, \end{aligned} \quad (6.35)$$

where Φ is an arbitrary function which can be decided by the initial condition. In Fig. 40 the change of slope profile obtained from equation (6.35) is shown. The initial slope is given by equation (6.21). In an early stage a gentle slope is formed around $x=4$ by the local delay of erosion. As the lower end of the gentle part becomes steep and erosive force acts intensely, even the hard rock is eroded. Consequently, the minimum point of erosion migrates on the slope upwards away from the hard part. The distortion of the profile nearly disappears at $t=0.5$. At the valley bed the knick point migrates upstream and may disappear in the long run.

In case B.4, a weak rock exists around $x=3$ in a manner expressed by

$$f_1(x, u) = \frac{20}{(x-3)^2 + 10}. \quad (6.36)$$

The change of slope in this case is shown in Fig. 41. In an early stage a convex slope is formed at the upper slope. The maximum point of erosion gradually migrates upwards away from the weak part.

In cases B.5 and B.6, the difference of the process of slope formation with the difference of the value of m' is considered.

From the value of $x^{m'}$ it is easily supposed that the larger the value of m' is, the faster proceeds the erosion at the lower part of the slope. Giving equation (6.21) as the initial condition, the solution of equation (6.7) is

$$u = -I_c(x - x_0) + u_0, \\ x_0 = \{x^{-(m'+1)} - K(m' - 1)t\}^{-1/(m' - 1)}. \quad (6.37)$$

The value of m' is set at 1.5 in case B.5 and at 0.5 in case B.6. The results are shown in Figs. 42 and 43.

In case B.5, slope degradation at the lower slope is rapid and the upper slope becomes convex. In case B.6, the amount of degradation is fairly uniform at most parts of the slope, and the slope degradation at the lower slope is slow. This may partly represent the process of valley bed degradation, since the value of m' is supposed to be smaller in the case. The rate of erosion in case of $m' = 1.5$ is about five times as great as in case $m' = 0.5$.

Finally it should be added that in the study of slope development, it must be avoided to discuss the validity of a model only by the comparison of theoretically derived profiles with actual slopes, because the initial conditions of actual slopes and the processes of slope deformation of them are unknown.

7. CONCLUDING REMARK

The mechanism of slope erosion and the process of slope development have been studied by physical and morphometrical analyses. Calculated values of erosion depth obtained from the theoretical equations of slope erosion agree quite well with the measured values obtained by morphometric measurements of volcanoes and coal slag heaps in Japan. Thus, the fundamental process of slope erosion and the change of valley bed can be clarified. Equations of slope development whose validity is demonstrated have been derived and general processes of slope development have been studied analytically.

In the present paper, the mechanism of erosion was considered under simple conditions, and it was tried to confirm the applicability of the equations by using the data obtained mainly from the morphometry of slopes and valleys which have favorable conditions. In addition, the equations derived here have some defects, and

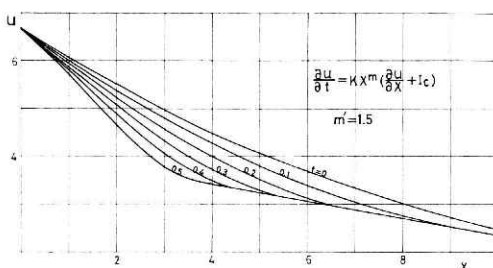


Fig. 42. Slope development in case B.5.
Difference of slope development with
the difference of m' ($m' = 1.5$).

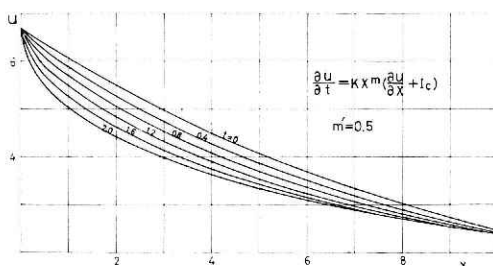


Fig. 43. Slope development in case B.6.
Difference of slope development with
the difference of m' ($m' = 0.5$).

further study should be done under more actual conditions, using the aids of field observations and experiments. Three dimensional analysis is also required to clarify the evolution of a drainage basin.

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浸食による山地斜面の発達

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山地斜面の浸食過程を、主要営力による浸食の物理的機構の検討、浸食地形の計測および斜面発達の式の数理解析により考察した。長時間の浸食過程においては、浸食に関係する多数の要因や浸食現象の効果が平均化されて、基本的、主導のプロセスが強く現われてくることが期待できるので、浸食の物理的機構を単純な条件の下で考察することができる。

山地斜面上の雨水流を等流状態にあるとして、容易に数量化できる地形要因を独立変数とする次の浸食の式Aを、水流の運動および連続の式、流砂量公式および流砂の連続式から導いた。

$$E = k_1 l^m \frac{d \sin^n \theta}{dl} + k_2 l^{m-1} \sin^n \theta. \quad (A式)$$

ここでE: 浸食深; l: 斜面長; θ : 斜面こう配; k_1, k_2, m, n : 定数。

マスマーブメントの2次浸食の場合についても、運動機構をきわめて単純化して、上式と同じ形をもつ式が得られた。長時間の平均状態において、斜面構成物質の除去は浸食力の大きさに比例するものとし、同時に安息堆積角の概念を導入して、次の浸食の式Bを導いた。

$$E = K l^{m'} (\sin \theta - \sin \theta_0)^{n'}. \quad (B式)$$

ここで θ_0 : 安息角; k, m', n' : 定数。

実斜面における浸食過程への浸食の式の適用性を、浸食地形の計測によって考察した。この際、原地形は浸食によって形成された谷を埋めることによって復元し、また広い範囲の斜面を二次元的に表わす計測方法によって、原地形および現地形の平均斜面形を得た。斜面上の各点の平均浸食深は、これら2種の斜面形の距離によって与えられる。

各種の規模および浸食ステージのものを含む日本の火山斜面において、B式から得られる平均浸食深の計算値は計測値と非常によく一致した。 k, m' および n' の値は最小二乗法によって決めた。相関係数は0.9以上で、すべて高度に有意の相関がある。ガリ浸食をうけている炭鉱のボタ山にもB式はよく適用できた。羊蹄山では岩質の差異による浸食の難易度および斜面方向の違いによる浸食強度の差を量的に得ることができた。 m', n' の値は、浸食ステージ、位置、規模などに無関係に、ほぼ1に近い値をとる。これは主導的な浸食営力の種類が同一であることを示す。 θ_0 の値は浸食が進むにつれて減少する。多くの火山斜面において、A式から得られる平均浸食深の計算値は計測値とよく一致した。これによって、実斜面においては、A式およびB式が示す物理的機構によって、基本的な浸食過程が進行していることが確かめられた。A式は合理的な形をもち一般的な適用性を示唆する。B式は物質連続の条件をみたしていないので、堆積現象などをよく示し得ない。火山放射谷の縦断形の変化もB式によって示すことができた。これらの結果から、谷底における浸食過程がもっとも主導的であって、谷壁の後退は横断面形の相似性をほぼ保ちながら谷底の低下量に応じて行なわれるものとすることによって、開析状態に関係なくB式が適用できる理由が説明される。常願寺川、多枝原谷の河床変動の測量値は、A式から得られる計算値とよい一致を示した。この谷では土石流が主要な浸食現象であることが認められている。以上の結果から、浸食の基本的プロセスは、浸食期間の長短や浸食営力の種類さらには火山性、非火山性などの斜面の性質とはあまり関係なく同一であると推定できる。

浸食過程のあるものは、運搬営力の介入がなく、斜面のこう配あるいは曲率の単独項によって示され则认为られている。これまでに提案されている斜面発達モデルの大部分は、これらの単独項から成立

っている。しかしながら、計測した斜面のいくつかについて行なった重回帰分析では、これらの単独項の有意性は全く認められなかった。これらは浸食の主導的なプロセスを示すものではないと推定される。

浸食の式の単純化と修正により、次の斜面発達式 A , B を導いた。

$$\frac{\partial u}{\partial t} = k_1 x^m \frac{\partial^2 u}{\partial x^2} + k_2 x^{m-1} \frac{\partial u}{\partial x}, \quad (A \text{ 式})$$

$$\frac{\partial u}{\partial t} = K x^{m'} \left(\frac{\partial u}{\partial x} + I_c \right). \quad (B \text{ 式})$$

ここで u : 高度; x : 斜面上端からの水平距離; t : 時間; I_c : 定数。

これらの式の妥当性は、2, 3 の計測斜面において、斜面低下量の計算値と計測値との一致によって確かめられた。

斜面発達式 A , B を各種の初期、境界条件の下で解くことによって、斜面形変化の一般的過程を考察した。一般に開析の初期には山麓において堆積が生ずるがやがて再開析が始まる。開析がある程度進むと、初期斜面の形状に無関係に共通した凹型斜面が形成されるが、これはほぼ対数曲線で示される。したがって、浸食地形の平均斜面形および谷床縦断形の平衡形は、ほぼ対数曲線であるといえる。山体の開析は初期には急速に進行するが、山体の半ばが消失するころから開析速度は急激に逓減する。日本の火山体は形成後 100 万年経過すると、山体の約 80% が浸食されると推定される。

山頂の凸型斜面の形成を、分水界近くでは掃流力 ≧ 限界掃流力が成立たないという条件を導入することによって A 式で記述できることを示した。 A 式を修正して、分水界が後退する場合のプロセスを示す斜面発達式を導いた。この式によって台地の開析と放射谷の発達の過程を示した。浸食係数を場所の関数とすることによって、地質条件に場所的な差がある場合の斜面形の変化を示した。